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Digital Filter Design

Chapter 2: Finite Impulse Response (FIR) Filter

2.1 Introduction

FIR filters are digital filters with finite impulse response. They are also known as **non-recursive digital filters** as they do not have the feedback (a recursive part of a filter), even though recursive algorithms can be used for FIR filter realization.

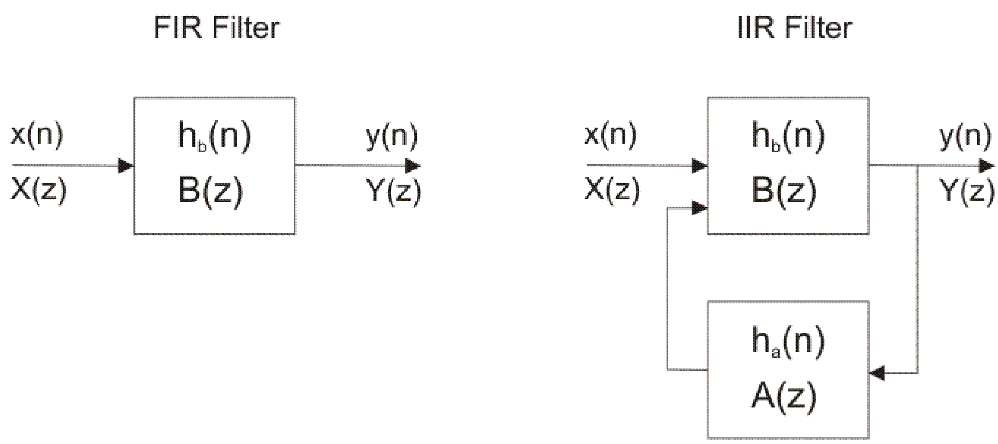
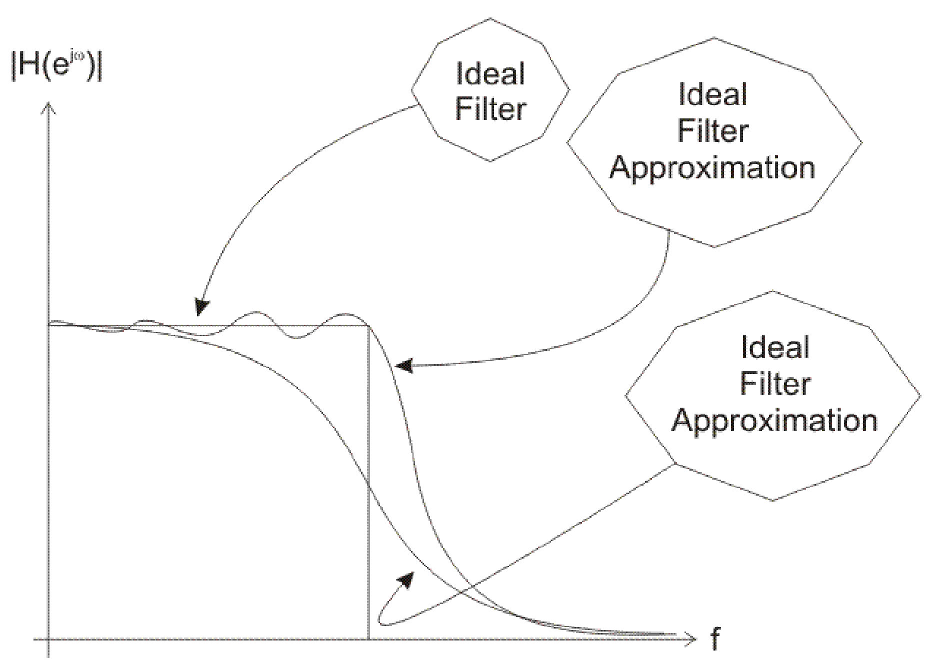


Figure 2-1-1. Block diagrams of FIR and IIR filters

FIR filters can be designed using different methods, but most of them are based on ideal filter approximation. The objective is not to achieve ideal characteristics, as it is impossible anyway, but to achieve sufficiently good characteristics of a filter. The transfer function of FIR filter approaches the ideal as the filter order increases, thus increasing the complexity and amount of time needed for processing input samples of a signal being filtered.



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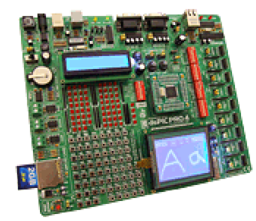
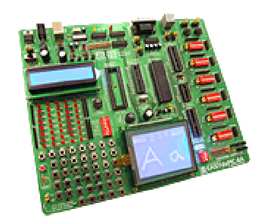
dsPICPRO4 Development System



The **dsPICPRO4** supports the latest 64- and 80-pin dsPIC30F microcontrollers from Microchip. It comes with a dsPIC30F6014A. The system includes an USB 2.0 programmer, mikroICD (In-Circuit Debugger), touch panel controller, CAN, RS485, DAC and RTC. [more info]

EasydsPIC4A Development System



EasydsPIC4A is a full-featured development system that supports the latest 18-, 28- and 40-pin dsPIC30F MCUs. It comes with the dsPIC30F4013 microcontroller. The board features a USB 2.0 programmer, mikroICD (In-Circuit Debugger), touch panel controller etc. [more info]

Figure 2-1-2. Ideal low-pass filter approximation

The resulting frequency response can be a monotone function or an oscillatory function within a certain frequency range. The waveform of frequency response depends on the method used in design process as well as on its parameters.

This book describes the most popular method for FIR filter design that uses window functions. The characteristics of the transfer function as well as its deviation from the ideal frequency response depend on the filter order and window function in use.

Each filter category has both advantages and disadvantages. This is the reason why it is so important to carefully choose category and type of a filter during design process.

FIR filters can have linear phase characteristic, which is not like IIR filters that will be discussed in Chapter 3. Obviously, in such cases when it is necessary to have a linear phase characteristic, FIR filters are the only option available. If the linear phase characteristic is not necessary, as is the case with processing speech signals, FIR filters are not good solution at all.

Figure 2-1-3 illustrates input and output signals of non-linear phase systems.

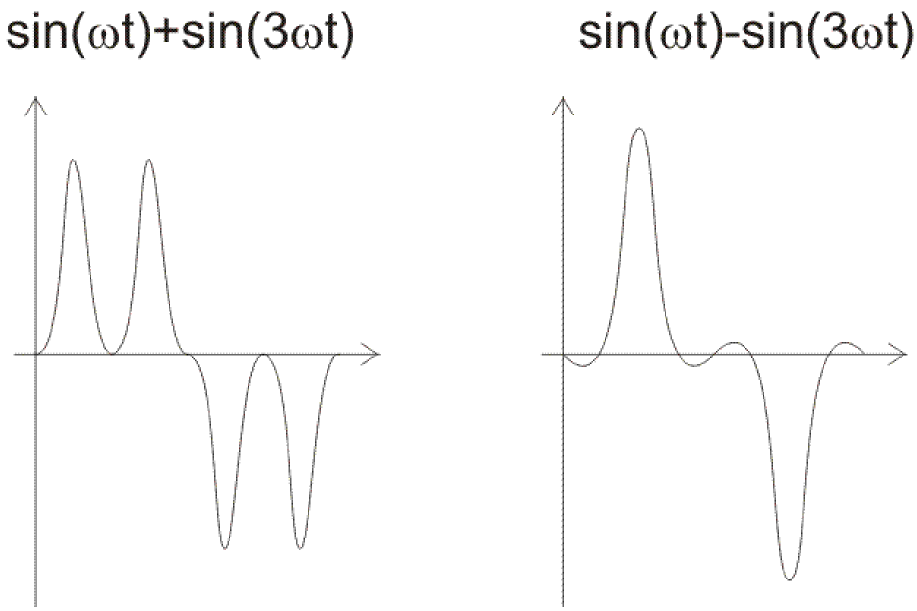


Figure 2-1-3. The effect of non-linear phase characteristic

The system introduces a phase shift of 0 radians at the frequency of ω, and π radians at three times that frequency. Input signal consists of natural frequency ω and one harmonic with the same amplitude at three times that frequency. Figure 2-1-3. shows the block diagram of input signal (left) and output signal (right). It is obvious that these two signals have different waveforms. The power of signals is not changed, nor the amplitudes of harmonics, only the phase of the second harmonic is changed.

If we assume that the input is a speech signal whose phase characteristic is not of the essence, such distortion in the phase of the signal would be unimportant. In this case, the system satisfies all necessary requirements. However, if the phase characteristic is of importance, such a great distortion mustn't be allowed.

In order that the phase characteristic of a FIR filter is linear, the impulse response must be symmetric or anti-symmetric, which is expressed in the following way:

$h[n] = h[N-n-1]$; symmetric impulse response (about its middle element)

$h[n] = -h[N-n-1]$; anti-symmetric impulse response (about its middle element)

One of the drawbacks of FIR filters is a high order of designed filter. The order of FIR filter is remarkably higher compared to an IIR filter with the same frequency response. This is the reason why it is so important to use FIR filters only when the linear phase characteristic is very important.

A number of delay lines contained in a filter, i.e. a number of input samples that should be saved for the purpose of computing the output sample, determines the order of a filter. For example, if the filter is assumed to be of order 10, it means that it is necessary to save 10 input samples preceeding the current sample. All eleven samples will affect the output sample of FIR filter.

The transform function of a typical FIR filter can be expressed as a polynomial of a complex variable z^{-1}. All the poles of the transfer function are located at the origin. For this reason, FIR filters are guaranteed to be stable, whereas IIR filters have potential to become unstable.

2.2 Finite impulse response (FIR) filter design methods

Most FIR filter design methods are based on ideal filter approximation. The resulting filter approximates the ideal characteristic as the filter order increases, thus making the filter and its implementation more complex.

The filter design process starts with specifications and requirements of the desirable FIR filter. Which method is to be used in the filter design process depends on the filter specifications and implementation. This chapter discusses the FIR filter design method using window functions.

Each of the given methods has its advantages and disadvantages. Thus, it is very important to carefully choose the right method for FIR filter design. Due to its simplicity and efficiency, the window method is most commonly used method for designing filters. The sampling frequency method is easy to use, but filters designed this way have small attenuation in the stopband.

As we have mentioned above, the design process starts with the specification of desirable FIR filter.

2.2.1 Basic concepts and FIR filter specification

First of all, it is necessay to learn the basic concepts that will be used further in this book. You should be aware that without being familiar with these concepts, it is not possible to understand analyses and synthesis of digital filters.

Figure 2-2-1 illustrates a low-pass digital filter specification. The word specification actually refers to the frequency response specification.

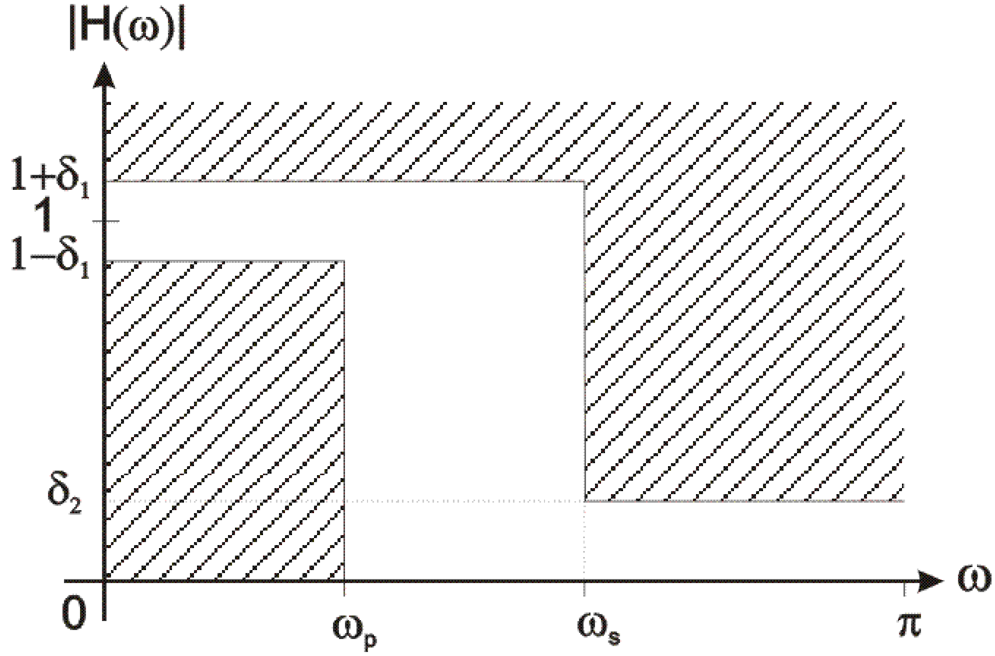


Figure 2-2-1a. Low-pass digital filter specification

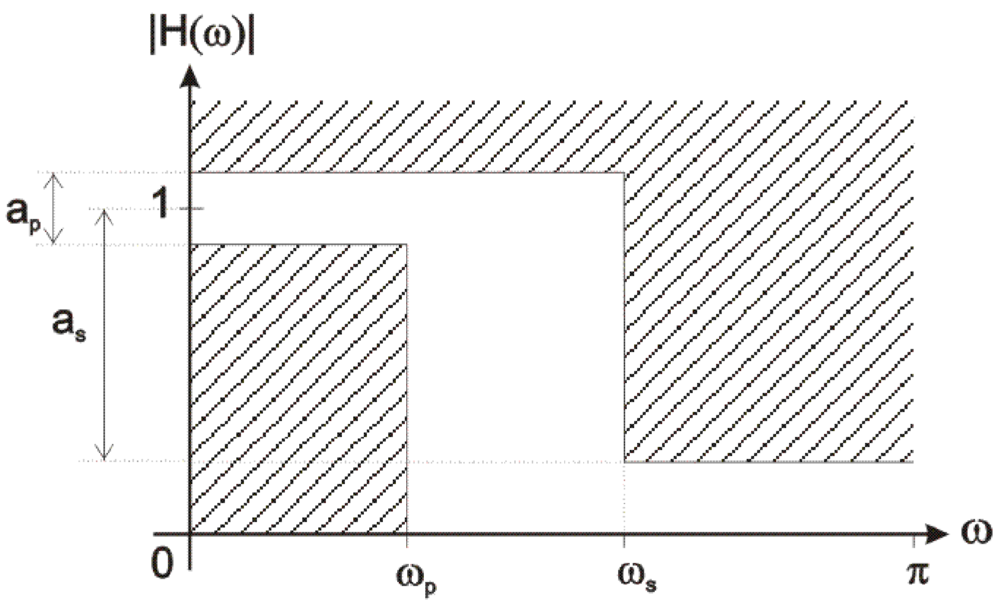


Figure2-2-1b. Low-pass digital filter specification

- ωp – normalized cut-off frequency in the passband;
- ωs – normalized cut-off frequency in the stopband;
- $\delta 1$ – maximum ripples in the passband;
- $\delta 2$ – minimum attenuation in the stopband **[dB]**;
- ap – maximum ripples in the passband; and
- as – minimum attenuation in the stopband **[dB]**.

$$a_p = 20\log_{10}\left(\frac{1+\delta_1}{1-\delta_1}\right)$$

$$a_s = -20\log_{10}\delta_2$$

Frequency normalization can be expressed as follows:

$$\omega = \frac{2\pi f}{f_s}$$

where:

- fs is a sampling frequency;
- f is a frequency to normalize; and
- ω is normalized frequency.

Specifications for high-pass, band-pass and band-stop filters are defined almost the same way as those for low-pass filters. Figure 2-2-2 illustrates a high-pass filter specification.

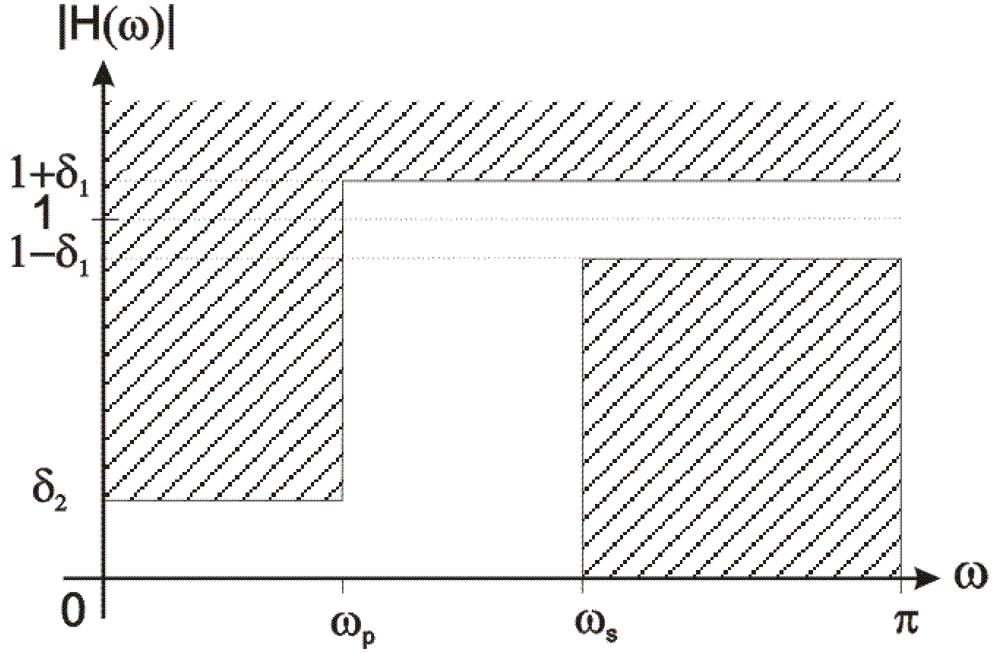


Figure 2-2-2a. High-pass digital filter specification

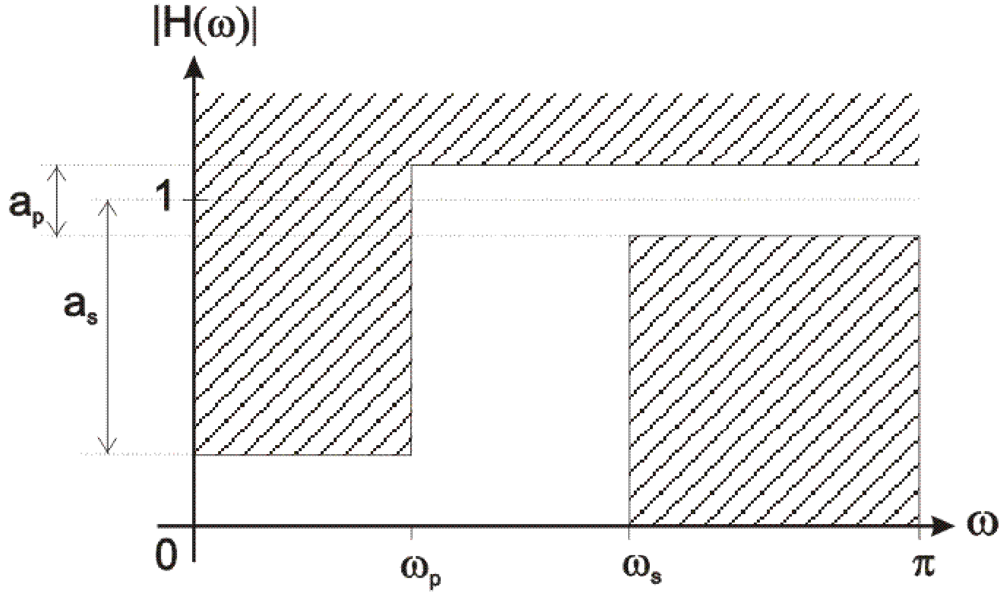


Figure 2-2-2b. High-pass digital filter specification

Comparing these two figures 2-2-1 and 2-2-2, it is obvious that low-pass and high-pass filters have similar specifications. The same values are defined in both cases with the difference that in the later case the passband is substituded by the stopband and vice versa.

Figure 2-2-3 illustrates a band-pass filter specification.

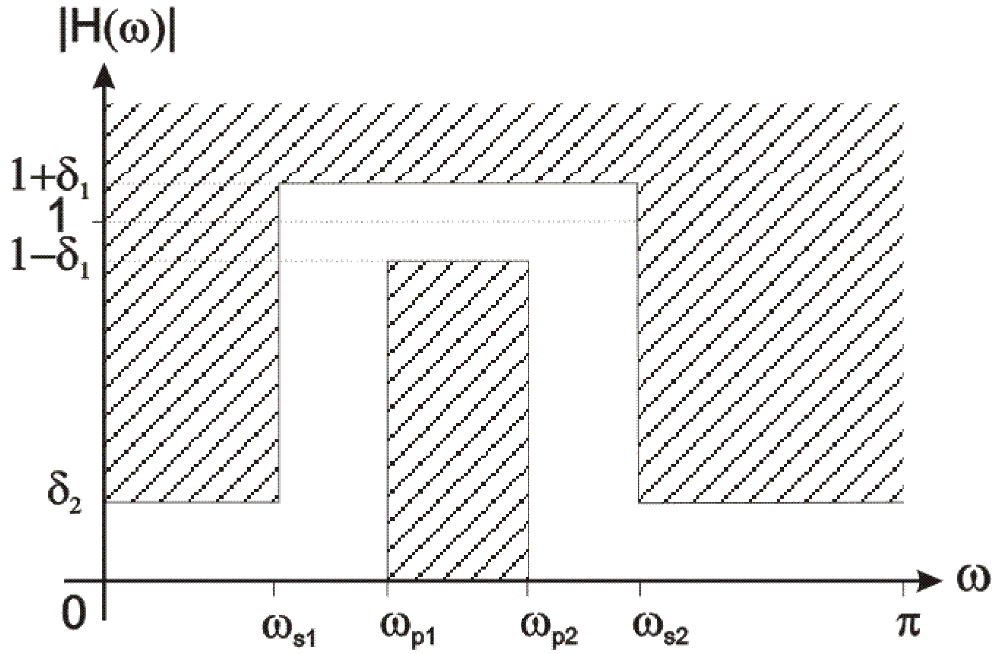


Figure 2-2-3a. Band-pass digital filter specification

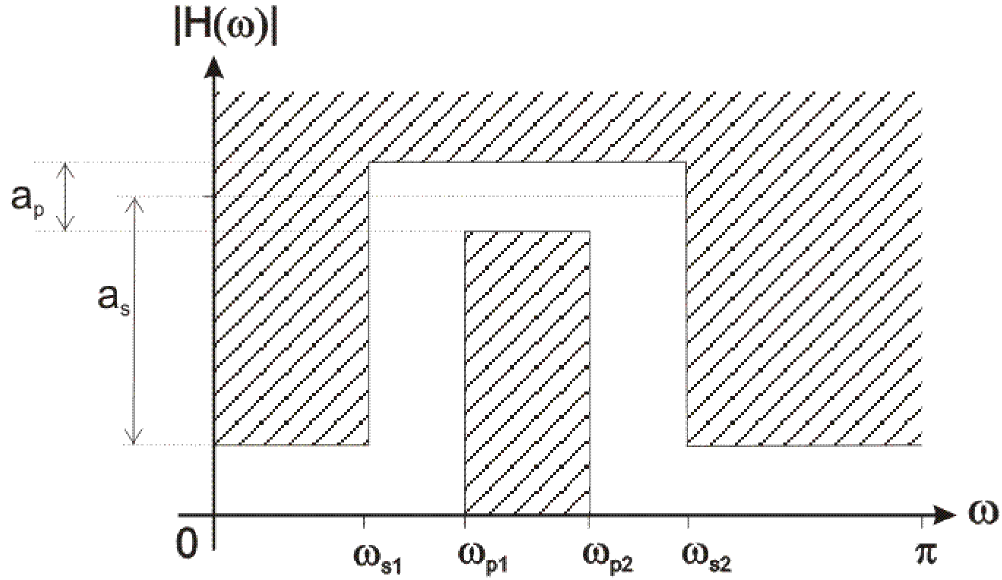


Figure 2-2-3b. Band-pass digital filter specification

Figure 2-2-4 illustrates a band-stop digital filter specification.

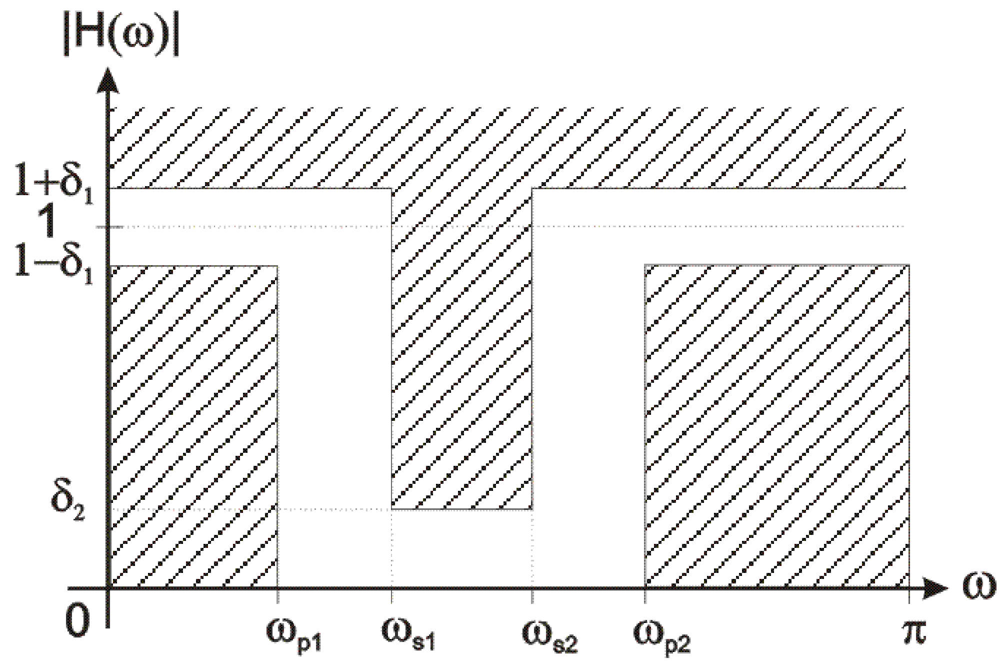


Figure 2-2-4a. Band-stop digital filter specification

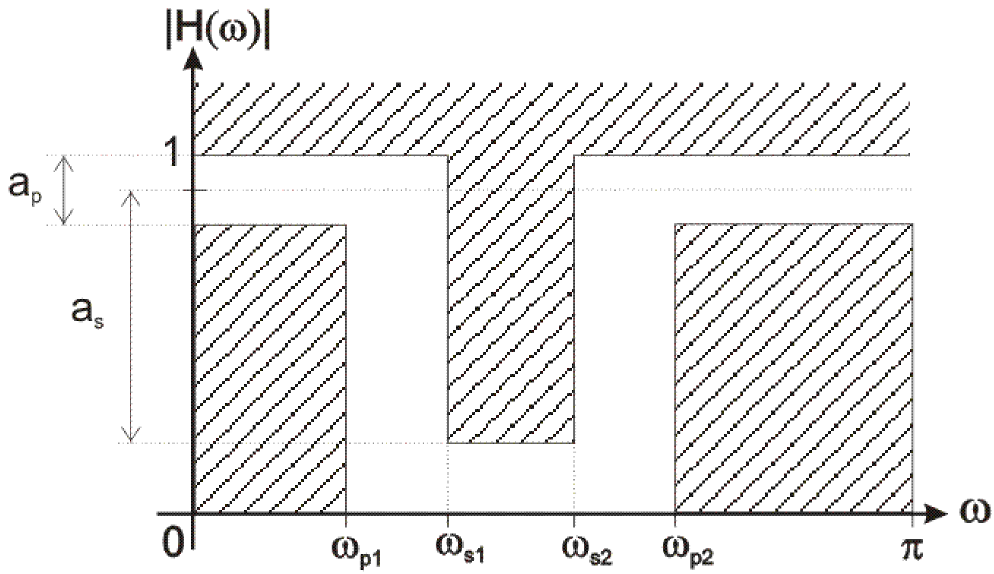


Figure 2-2-4b. Band-stop digital filter specification

2.2.2 Z-transform

The Z-transform is performed upon discrete-time signals. It converts a discrete time-domain signal into a complex frequency-domain representation. It is very suitable for analysing discrete time-domain signals and systems. The z-transform is derived from the Fourier discrete time-domain transformation and is considered the basic operation in digital filter design process.

The Z-transform is defined as:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

where **z** is the complex number.

Example:

Assume that samples of a discrete-time signal x(n) are known. It is necessary to transform this signal through the ztransform and Fourier fransform.

$$x(n)=\{1,2,3,4,5,4,3,2,1\} \; ; \; 0 \le n \le 8$$

z-transform is defined as follows:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

It becomes:

$$X(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 5z^{-4} + 4z^{-5} + 3z^{-6} + 2z^{-7} + z^{-8}$$

$$X(z) = \frac{z^8 + 2z^7 + 3z^6 + 4z^5 + 5z^4 + 4z^3 + 3z^2 + 2z + 1}{z^8}$$

The last expression represents the z-transform of the given signal.

The Fourier transform can be found by rewriting the previous expression in terms of z as z=ejω. It further becomes:

$$X(e^{j\omega}) = \frac{e^{j8\omega} + 2e^{j7\omega} + 3e^{j6\omega} + 4e^{j5\omega} + 5e^{j4\omega} + 4e^{j3\omega} + 3e^{j2\omega} + 2e^{j\omega} + 1}{e^{j8\omega}}$$

Figure 2-2-5 illustrates the frequency spectrum of the given signal.

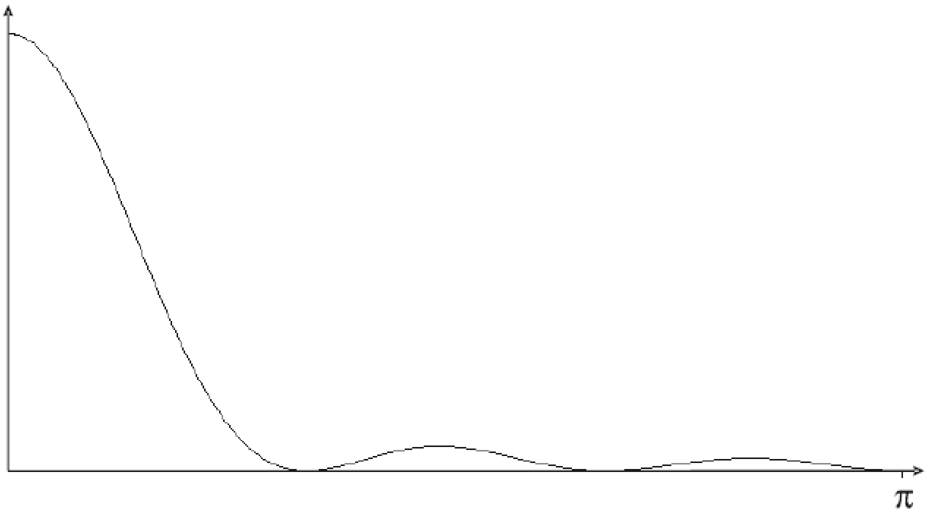


Figure 2-2-5. Frequency spectrum of the given signal

Comparing Z and Fourier transforms, it is easy to notice some similarities between them:

$$X\left(e^{j\omega}\right)=\sum_{n=-\infty}^{\infty}x(n)e^{-j\omega}=X\left(z=e^{j\omega}\right)$$

In polar coordinate system, the complex number z may be expressed as:

$$z=re^{j\omega}$$

The two last expressions lead us to the conclusion that the Fourier transform is just a special form of the z-transform for r=1.
In the z plane, the Fourier transform is represented as a unit circle, which can be seen in Figure 2-2-6 below.

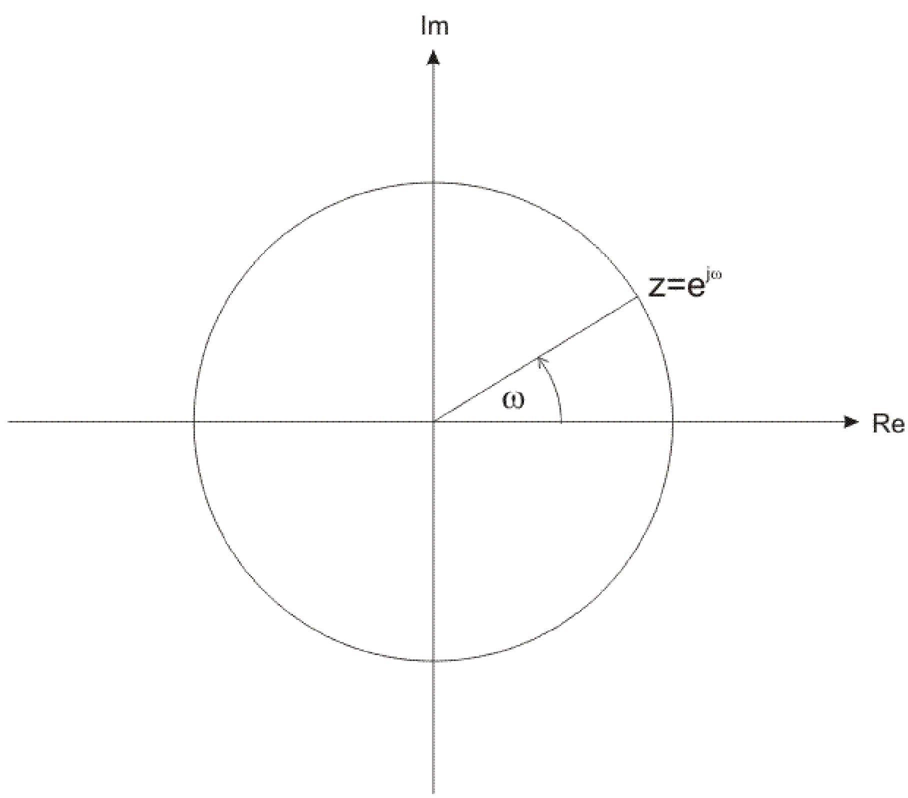


Figure 2-2-6. Fourier transform in the z plane

2.2.2.1 Transform function of discrete-time systems

The Z-transform is primarily used for finding the transfer function of linear discrete-time systems. When the transfer function is found, it is necessary to consider its zeros and poles in the z plane. The transfer function of discrete-time systems is defined as:

$$H(z) = \frac{\sum_{i=0}^{M-1} b_i z^{-i}}{\sum_{j=0}^{N-1} a_j z^{-j}} = H_0 \frac{\prod_{i=0}^{M-1} (1 - q_i z^{-i})}{\prod_{j=0}^{N-1} (1 - p_j z^{-j})}$$

where:

- bi are the feedforward filter coefficients (non-recursive part);
- aj are the feedback filter coefficients (recursive part);
- H0 is a constant;
- qi are the zeros of the transfer function; and
- pj are the poles of the transfer function.

The recursive part of the transfer function is actually a feedback of discrete-time system. As we previously mentioned, FIR filters do not have this recursive part of the transfer function, so the expression above can be simplified in the following way:

$$H(z) = \sum_{i=0}^{M-1} b_i z^{-i} = H_0 \prod_{i=0}^{M-1} (1 - q_i z^{-i})$$

The impulse response of discrete-time system is obtained from inverse z-transform of the transfer function i.e. the transfer function of discrete-time system is actually the Z-transform of impulse response:

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$$

where h(n) is the impulse response of discrete-time system.

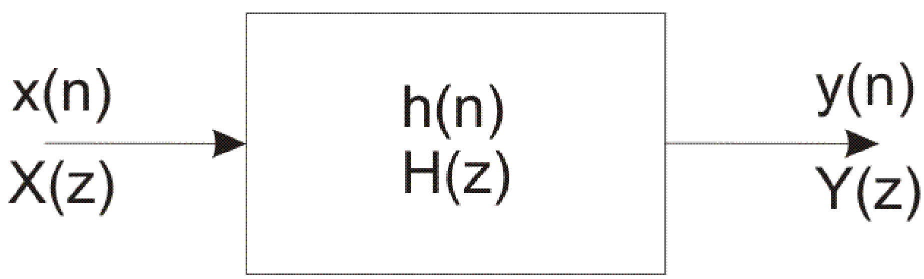


Figure 2-2-7. Block diagram of a linear discrete-time system

In the time domain, the discrete-time system, shown in Figure 2-2-7, can also be expressed as the convolution of the input signal **x(n)** and the impulse response **h(n)** of the system:

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

In the frequency domain, the discrete-time system, shown in Figure 2-2-7, can be expressed as the multiplication of the Z-transform input signal **X(z)** and the transfer function **H(z)** of the system:

$$Y(z) = X(z) H(z)$$

which further gives:

$$H(z) = \frac{Y(z)}{X(z)}$$

The first way of representing discrete-time systems is more suitable for software implementation itself, whereas the later is more suitable for analyse, hardware implementation (described later) and synthesis, i.e. discrete-time system design.

Example:

The impulse response of a 10-th order FIR filter designed using the Hamming window (discussed in the next chapter) is:

h(n) = {0, - 0.0127, - 0.0248, 0.0638, 0.2761, 0.4, 0.2761, 0.0638, - 0.0248, - 0.0127, 0}

The transfer function of this filter is found via the ztransform of impulse response:

$$H(z) = \sum_{n=0}^{10} h(n)z^{-n} = 0 - 0.0127z^{-1} - 0.0248z^{-2} + 0.0638z^{-3} + 0.2761z^{-4} + 0.4z^{-5}$$

$$+ 0.2761z^{-6} + 0.0638z^{-7} - 0.0248z^{-8} - 0.0127z^{-9} + 0z^{-10}$$

$$= -0.0127z^{-1} - 0.0248z^{-2} + 0.0638z^{-3} + 0.2761z^{-4} + 0.4z^{-5}$$

$$+ 0.2761z^{-6} + 0.0638z^{-7} - 0.0248z^{-8} - 0.0127z^{-9}$$

Using the following expression:

$$X\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega} = X\left(z = e^{j\omega}\right)$$

it is possible to yield the transfer function of the fixed normalized frequency. If for example $\omega = 0.2\pi$ then:

$$H(e^{j0.2\pi}) = -0.0127e^{-j0.2\pi} - 0.0248z^{-j0.4\pi} + 0.0638z^{-j0.6\pi} + 0.2761z^{-0.8\pi} + 0.4z^{-j\pi}$$

$$+ 0.2761z^{-j1.2\pi} + 0.0638z^{-j1.4\pi} - 0.0248z^{-j1.6\pi} - 0.0127z^{-j1.8\pi}$$

$$|H(e^{j0.2\pi})| = (-0.0127\cos(-0.2\pi) - 0.0248\cos(-0.4\pi) + 0.0638\cos(-0.6\pi) + 0.2761\cos(-0.8\pi)$$

$$+ 0.4\cos(-\pi)$$

$$+ 0.2761\cos(-1.2\pi) + 0.0638\cos(-1.4\pi) - 0.0248\cos(-1.6\pi) - 0.0127\cos(-1.8\pi))^2 +$$

$$(-0.0127\sin(-0.2\pi) - 0.0248\sin(-0.4\pi) + 0.0638\sin(-0.6\pi) + 0.2761\sin(-0.8\pi)$$

$$+ 0.4\sin(-\pi)$$

$$+ 0.2761\sin(-1.2\pi) + 0.0638\sin(-1.4\pi) - 0.0248\sin(-1.6\pi) - 0.0127\sin(-1.8\pi))^2$$

$$= (-0.92205)^2 + 0^2 = 0.92205$$

One example of hardware realization of this filter is illustrated in Figure 2-2-8.

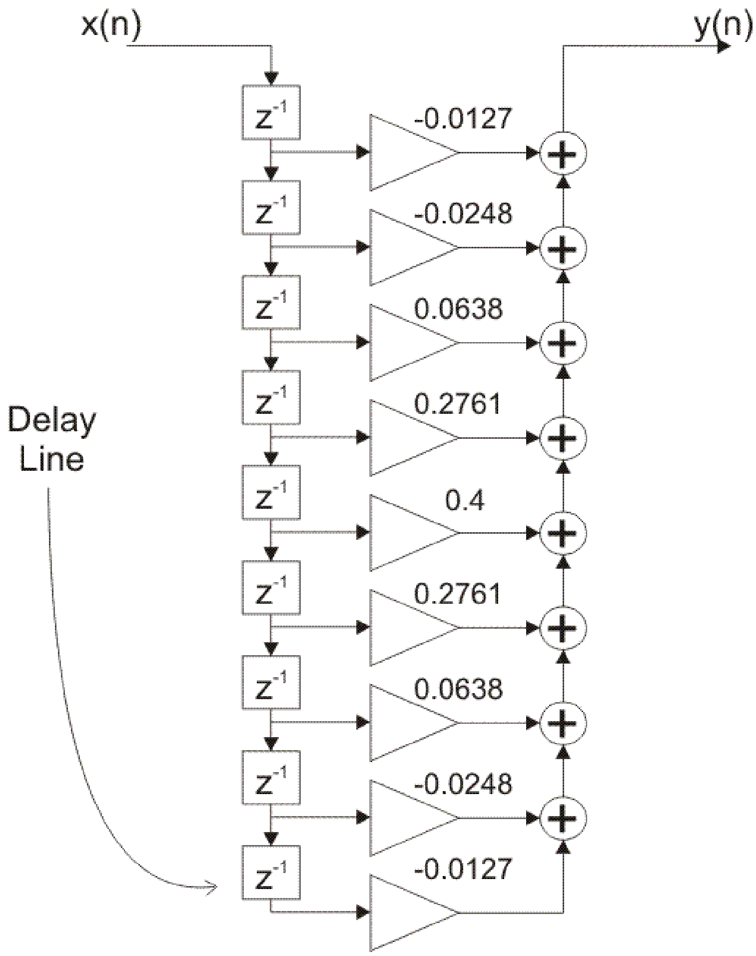


Figure 2-2-8. FIR filter realization

Software realization requires a buffer of minimum length 9. Buffers are usually circular and their length can be expressed as 2^n, which in this case means that the circular buffer is of length 16=2^4.

2.2.2.2 Effect of the poles and zeros of the transfer function

The location of zeros and poles of the transfer function is very important for discretetime system analyses and synthesis. According to their position it is possible to test stability of a discrete-time system, detect round-off errors made due to software implementation of a filter as well as errors in the coefficients encountered during hardware implementation of a filter.

In order that a discrete-time system is stable, all poles of the discrete-time system transfer function must be located inside the unit circle, as shown in Figure 2-2-6. If this requirement is not satisfied, the system becomes unstable, which is very dangerous. The location of zeros doesn't affect the stabilty of discrete-time systems. Recalling that FIR flters do not have a feedback, which further means that the transfer function has no poles. This causes a FIR filter to be always stable. Filter stability will be discussed in more details along with IIR filters which have potential to become unstable because of the feedback they have. This property of FIR filters actually represents their essential adventage. From now on, only the zeros of the transfer function will be discussed in this chapter.

An error in coefficient representation is always produced due to software and hardware implementation. In software implementation, an error is triggered by the finite word-length effect, whereas in hardware implementation, it ocurrs due to impossibility of representing the coefficients with absolute accuracy. The result in both cases is that the value of coefficients differs from their value obtained in design process. Such errors cause frequency deviation of discrete-time system designed.

Frequency deviation depends on the spacing between the zeros of the FIR filter transfer function. FIR filter coefficient error affects more the frequency characteristic as the spacing between the zeros of the transfer function narrows. This property is particularly typical of highorder filters because their zeros are very close each other. However, slight errors in coefficient representation may cause large frequency deviations.

Figure 2-2-9 illustrates the required and obtained frequency characteristic of a FIR filter. The finite word-length effect on the transform function of a FIR filter is clearly marked. Assume that a 50-th order low-pass FIR filter with normalized cut-off frequency of 0.25 Hz is designed using the Hann window.

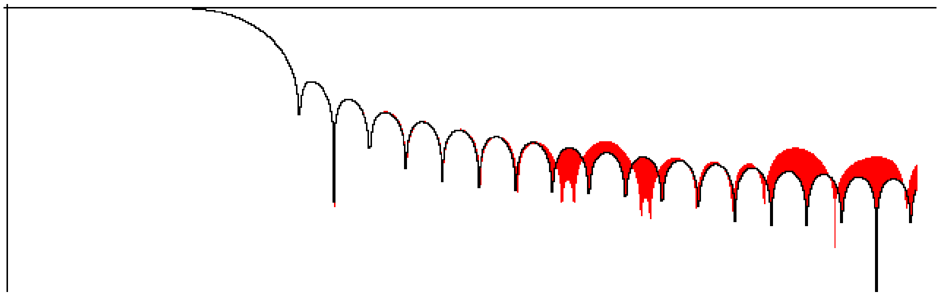


Figure 2-2-9. Deviation from required frequency characteristic

The frequency deviation shown in Figure 2-2-9 is basically slight deviation, even though it is very large at certain points. The minimum attenuation and the width of transition region of the resulting IIR filter remain unchanged, so that such deviation is acceptable. However, as this is not a common case, it is necessary to be very careful when designing high order filters because the transfer function zeros get closer, while more affecting the resulting frequency characteristic.

2.2.3 Ideal filter approximation

The ideal filter frequency response is used when designing FIR filters using window functions. The objective is to compute the ideal filter samples. FIR filters have finite impulse response, which means the ideal filter frequency sampling must be performed in a finite number of points. As the ideal filter frequency response is infinite, it is easy to produce sampling errors. The error is less as the filter order increases.

Figure 2-2-10 illustrates the transfer functions of four standard ideal filters.

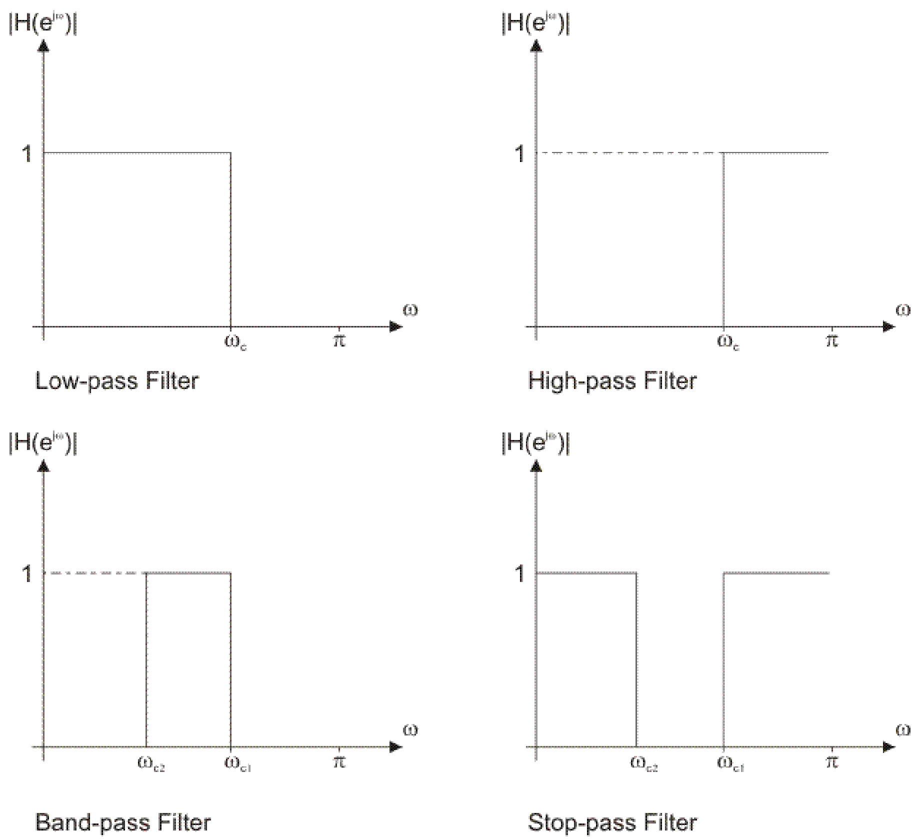


Figure 2-2-10. Transfer functions of four standard ideal filters

The ideal filter frequency response can be computed via inverse Fourier transform. The four standard ideal filters frequency responses are contained in the table 2-2-1 below.

Type of filter	Frequency response $h_d[n]$
low-pass filter	$h_d[n] = \begin{cases} \dfrac{\sin[\omega_c(n-M)]}{\pi(n-M)}; & n \neq M \\ \dfrac{\omega_c}{\pi}; & n = M \end{cases}$
high-pass filter	$h_d[n] = \begin{cases} 1-\dfrac{\omega_c}{\pi}; & n \neq M \\ -\dfrac{\sin(\omega_c(n-M))}{\pi(n-M)}; & n = M \end{cases}$
band-pass filter	$h_d[n] = \begin{cases} \dfrac{\sin(\omega_{c2}(n-M))}{\pi(n-M)} - \dfrac{\sin(\omega_{c1}(n-M))}{\pi(n-M)}; & n \neq M \\ \dfrac{\omega_{c2}-\omega_{c1}}{\pi}; & n = M \end{cases}$
band-stop filter	$h_d[n] = \begin{cases} \dfrac{\sin(\omega_{c1}(n-M))}{\pi(n-M)} - \dfrac{\sin(\omega_{c2}(n-M))}{\pi(n-M)}; & n \neq M \\ 1-\dfrac{\omega_{c2}-\omega_{c1}}{\pi}; & n = M \end{cases}$

Table 2-2-1. The frequency responses of four standard ideal filters

The value of variable **n** ranges between 0 and N, where N is the filter order. A constant M can be expressed as M = N / 2. Equivalently, N can be expressed as N = 2M.

The constant M is an integer if the filter order N is even, which is not the case with odd order filters. If M is an integer (even filter order), the ideal filter frequency response is symmetric about its Mth sample which is found via expression shown in the table 2-2-1 above. If M is not an integer, the ideal filter frequency response is still symmetric, but not about some frequency response sample.

Since the variable **n** ranges between 0 and N, the ideal filter frequency response has N+1 sample.

If it is needed to find frequency response of a non-standard ideal filter, the expression for inverse Fourier transform must be used:

$$h_d[n] = \frac{1}{\pi}\int_0^{\pi} e^{j\omega(n-M)}d\omega$$

Non-standard filters are rarely used. However, if there is a need to use some of them, the integral above must be computed via various numerical methodes.

2.2.4 FIR filter design using window functions

The FIR filter design process via window functions can be split into several steps:

1. Defining filter specifications;
2. Specifying a window function according to the filter specifications;
3. Computing the filter order required for a given set of specifications;
4. Computing the window function coefficients;
5. Computing the ideal filter coefficients according to the filter order;
6. Computing FIR filter coefficients according to the obtained window function and ideal filter coefficients;
7. If the resulting filter has too wide or too narrow transition region, it is necessary to change the filter order by increasing or decreasing it according to needs, and after that steps 4, 5 and 6 are iterated as many times as needed.

The final objective of defining filter specifications is to find the desired normalized frequencies (ωc, ωc1, ωc2), transition width and stopband attenuation. The window function and filter order are both specified according to these parameters.

Accordingly, the selected window function must satisfy the given specifications. This point will be discussed im more detail in the next chapter (2.3).

After this step, that is, when the window function is known, we can compute the filter order required for a given set of specifications. One of the techniques for computing is provided in chapter 2.3.

When both the window function and filter order are known, it is possible to calculate the window function coefficients w[n] using the formula for the specified window function. This issue is also covered in the next chapter.

After estimating the window function coefficients, it is necessary to find the ideal filter frequency samples. The expressions used for computing these samples are discussed in section 2.2.3 under **Ideal filter approximation**. The final objective of this step is to obtain the coefficients hd[n]. Two sequencies w[n] and hd[n] have the same number of elements.

The next step is to compute the frequency response of designed filter h[n] using the following expression:

$$h[n] = w[n] \cdot h_d[n]$$

Lastly, the transfer function of designed filter will be found by transforming impulse response via Fourier transform:

$$H(e^{j\omega}) = \sum_{n=0}^{N} h[n] \cdot e^{-jn\omega}$$

or via Z-transform:

$$H(z) = \sum_{n=0}^{N} h[n] z^{-n}$$

If the transition region of designed filter is wider than needed, it is necessary to increase the filter order, reestimate the window function coefficients and ideal filter frequency samples, multiply them in order to obtain the frequency response of designed filter and reestimate the transfer function as well. If the transition region is narrower than needed, the filter order can be decreased for the purpose of optimizing hardware and/or software resources. It is also necessary to reestimate the filter frequency coefficients after that. For the sake of precise estimates, the filter order should be decreased or increased by 1.

2.2.5 FIR filter realization

FIR filter transfer function can be expressed as:

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{n=0}^{N-1} h[n] \cdot z^{-n}$$

The frequency response realized in the time domain is of more interest for FIR filter realization (both hardware and software). The transfer function can be found via the z-transform of a FIR filter frequency response. FIR filter output samples can be computed using the following expression:

$$y[n] = \sum_{k=0}^{N-1} h[k] \cdot x[n-k]$$

where:

- **x[k]** are FIR filter input samples;
- **h[k]** are the coefficients of FIR filter frequency response; and
- **y[n]** are FIR filter output samples.

A good property of FIR filters is that they are less sensitive to the accuracy of constants than IIR filters of the same order.

There are several types of FIR filter realization. This chapter covers direct, direct transpose and cascade realizations which are very convenient for the hardware implementation of a filter. As for the software implementation, direct and optimized realizations will be discussed in this book.

2.2.5.1 Direct realization

Direct realization of FIR filter is based on the direct implementation of this expression:

$$y[n] = \sum_{k=0}^{N-1} h[k] \cdot x[n-k]$$

Direct realization is also known as a transversal filter.

Figure 2-2-11 illustrates the block diagram describing the hardware direct realization of a FIR filter.

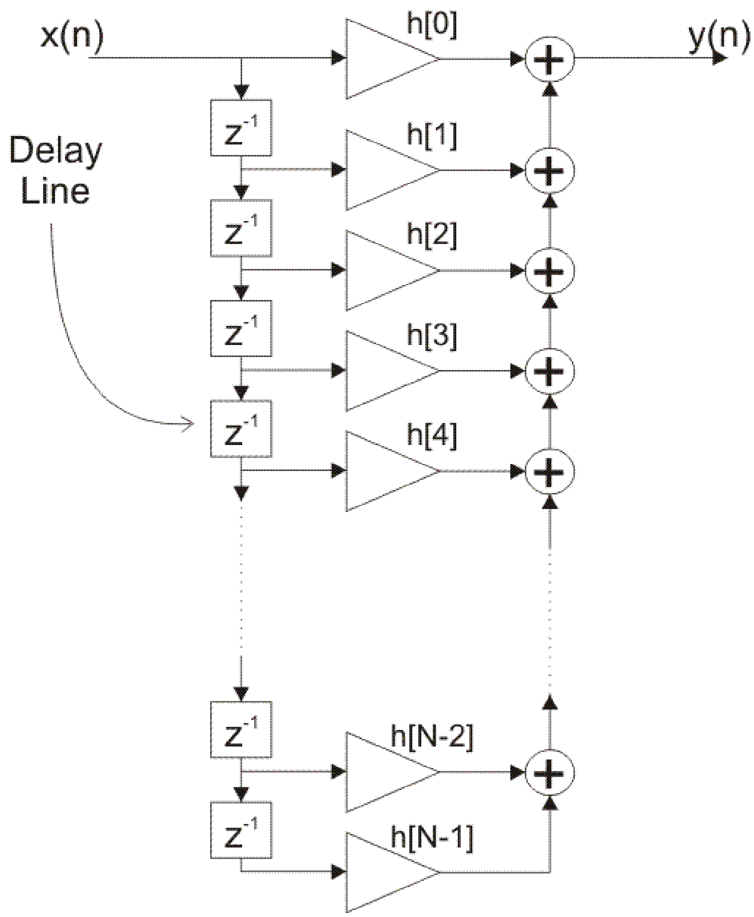


Figure 2-2-11. FIR filter direct realization

For direct realization structure, the multiplication constants are the same as the transfer function coefficients, i.e. the FIR filter frequency response coefficients.

As for software direct realization of the FIR filter, it is necessary to provide a buffer for minimum N samples, where N is the number of FIR filter coefficients. For its simplicity and speed, most commonly used buffer is so called circular buffer the length of which can be expressed as 2^k. The value of the constant k is a minimum value for which the expression N ≤ 2^k is valid. Accordingly:

$$k = \lfloor 1 + \log_{(2)} N \rfloor$$

where the operator $\lfloor \ \rfloor$ represents rounding down to a less value.

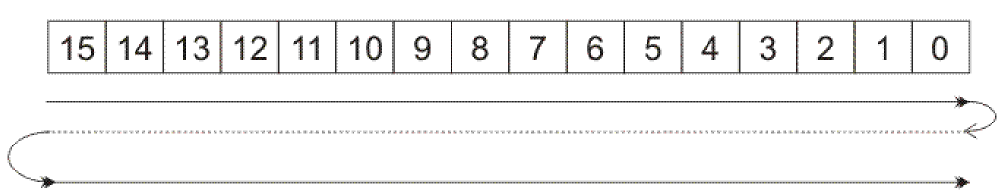


Figure 2-2-12. Circular buffer of length 16 = 2^4

The algorithm used for software direct realization of FIR filter consists of:

1. Reading the samples of a signal being filtered;
2. Storing a new sample on the first available location; and
3. Performing a convolution operation upon filter coefficients (frequency response coefficients), resulting in a FIR filter output sample.

Since the buffer is 16 bits wide, circular buffer addressing is performed using addressing mode 16:

$$(5+1)_{\text{mode }16}=6$$

$$(15+1)_{\text{mode }16}=0$$

Example: Assuming that a filter used in this example is a 5th order FIR filter. Design this filter using direct realization with circular buffer. The buffer length needs to be 2^k.

The buffer length is obtained in the following way:

$$k = \lfloor 1+\log_{(2)}N \rfloor = \lfloor 1+\log_{(2)}5 \rfloor = \lfloor 3.322 \rfloor = 3$$

This means that the minimum length of circular buffer is 2^3 = 8.

The contents of the buffer after receiving the first 10 samples is shown in the table 2-2-2. Input samples are denoted by x[n], whereas the shaded cells denote buffer locations that have been changed.

STEP	ADDR. 7	ADDR. 6	ADDR. 5	ADDR. 4	ADDR. 3	ADDR. 2	ADDR. 1	ADDR. 0
0								
1								x[0]
2							x[1]	x[0]
3						x[2]	x[1]	x[0]
4					x[3]	x[2]	x[1]	x[0]
5				x[4]	x[3]	x[2]	x[1]	x[0]
6			x[5]	x[4]	x[3]	x[2]	x[1]	x[0]
7		x[6]	x[5]	x[4]	x[3]	x[2]	x[1]	x[0]
8	x[7]	x[6]	x[5]	x[4]	x[3]	x[2]	x[1]	x[0]
9	x[7]	x[6]	x[5]	x[4]	x[3]	x[2]	x[1]	x[8]
10	x[7]	x[6]	x[5]	x[4]	x[3]	x[2]	x[9]	x[8]

Table 2-2-2. Input circular buffer after receiving 10 samples

For software realization, filtering of input samples is performed as per formula below:

$$y[n] = \sum_{k=0}^{N-1} h[k] \cdot x[n-k]$$

2.2.5.2 Direct transpose realization

Direct transpose realization is similar to direct realization in many ways. Speaking about software implementation, the direct transpose realization must also have a buffer of minimum length N, where N is the number of FIR filter coefficients.

Figure 2-2-13 illustrates the block diagram describing hardware direct transpose realization of a FIR filter.

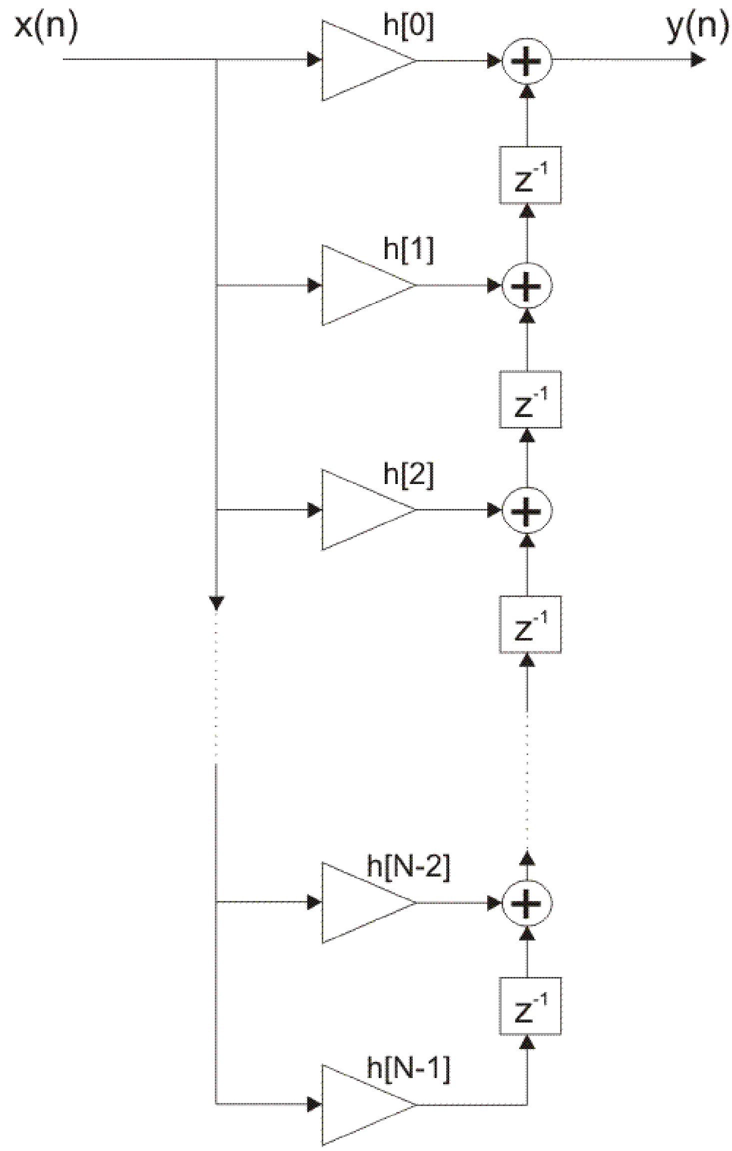


Figure 2-2-13. Direct transpose realization of a FIR filter

There are no significant differences between direct and direct transpose realizations. Both structures have the same number of delay elements, the same number of multipliers and the same coefficients to perform multiplication upon.

2.2.5.3 Cascade realization

Cascade realization, very convenient for its modular structure, is commonly used for FIR filter hardware realization. When using this realization, a filter is divided into several low-order sections. The second-order sections are most commonly used. Individual sections are mostly in direct form realization, although they can also be in direct transpose form realization. The cascade realization is normally used for higher order filter realization.

The transfer function of the cascade realization looks as follows:

$$H(z) = h[0] \cdot \prod_{k=1}^{M} \left(1 + a_{k1} \cdot z^{-1} + a_{k2} \cdot z^{-2}\right)$$

where:

- M is the number of sections; and
- ak1, ak2 are the multiplication coefficients of section k.

Figure 2-2-14 illustrates the block diagram describing the hardware cascade realization of a FIR filter.

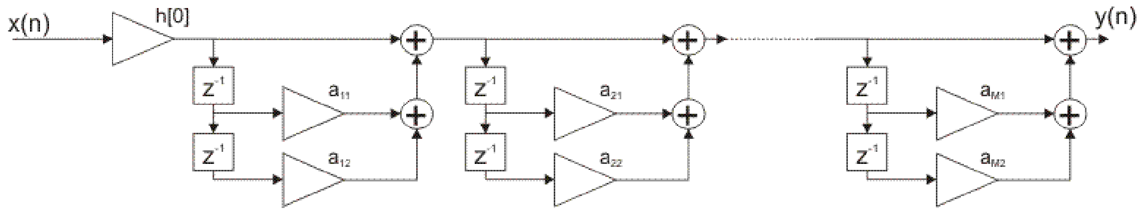


Figure 2-2-14. FIR filter cascade realization

The number of multipliers, adders and delays is the same as for direct realization. The main advantage of the cascade realization is its modularity, otherwise very convenient for hardware implementation. The cascade of second order sections is important for the realization of the filters of arbitrary order.

2.2.5.4 Optimized realization

Optimized realization has less, but more demanding multipliers for realization. This realization is most commonly used for software implementation of FIR filters, because a reduction in the number of multipliers enhances the process of convolution (samples filtering process).

Optimized realization utilizes the symmetry of frequency response coefficients. There are also anti-symmetric FIR filters that are beyond the scope of this book. Anyway, the optimized realization may be used for these filters as well.

The symmetry of the coefficients of FIR filter frequency response can be expressed by equation below:

$$h[k] = h[N-1-k]$$

This symmetry makes it possible for the transfer function to be expressed as follows:

$$H(z) = h[0]\cdot(1+z^{-N+1}) + h[1]\cdot(z^{-1}+z^{-N+2}) + \ldots + h[N\backslash 2-1]\cdot(z^{-N\backslash 2+1}+z^{-N\backslash 2}) \quad \text{for even N}$$

$$H(z) = h[0]\cdot(1+z^{-N+1}) + h[1]\cdot(z^{-1}+z^{-N+2}) + \ldots + h[(N-1)\backslash 2]\cdot z^{-(N-1)\backslash 2} \quad \text{for odd N}$$

Figure 2-2-15 illustrates the block diagram of optimized realization for even N, whereas Figure 2-2-16 illustrates that for odd N.

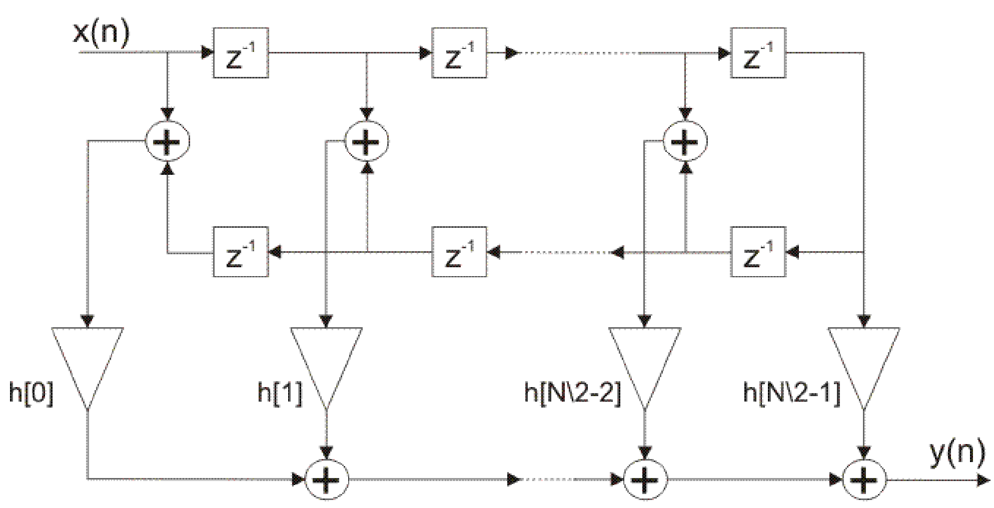


Figure 2-2-15. Optimized realization for odd frequency response

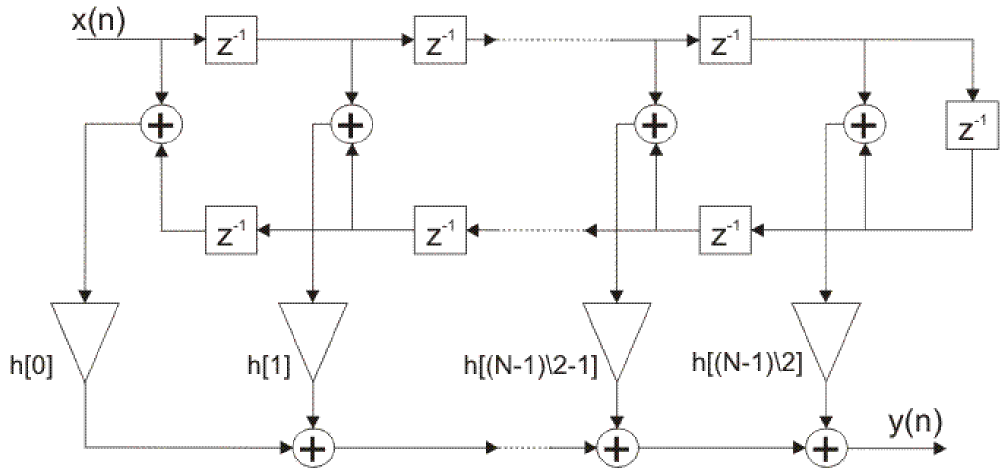


Figure 2-2-16. Optimized realization for even frequency response

2.3 Window functions

The window method is most commonly used method for designing FIR filters. The simplicity of design process makes this method very popular.

A window is a finite array consisting of coefficients selected to satisfy the desirable requirements. This chapter provides a few methods for estimating coefficients and basic characteristics of the window itself as well as the result filters designed using these coefficients. The point is to find these coefficients denoted by w[n].

When designing digital FIR filters using window functions it is necessary to specify:

- A window function to be used; and
- The filter order according to the required specifications (selectivity and stopband attenuation).

These two requirements are interrelated. Each function is a kind of compromise between the two following requirements:

- The higher the selectivity, i.e. the narrower the transition region; and
- The higher suppression of undesirable spectrum, i.e. the higher the stopband attenuation.

Table 2-3-1 below contains all window functions mentioned in this chapter and briefly compares their selectivity and stopband attenuation.

WINDOW FUNCTION	NORMALIZED LENGTH OF THE MAIN LOBE FOR N=20	TRANSITION REGION FOR N=20	MINIMUM STOPBAND ATTENUATION OF WINDOW FUNCTION	MINIMUM STOPBAND ATTENUATION OF DESIGNED FILTER
Rectangular	0.1π	0.041π	13 dB	21 dB
Triangular (Bartlett)	0.2π	0.11π	26 dB	26 dB
Hann	0.21π	0.12π	31 dB	44 dB
Bartlett-Hanning	0.21π	0.13π	36 dB	39 dB
Hamming	0.23π	0.14π	41 dB	53 dB
Bohman	0.31π	0.2π	46 dB	51 dB
Blackman	0.32π	0.2π	58 dB	75 dB
Blackman-Harris	0.43π	0.32π	91 dB	109 dB

Table 2-3-1. Comparison of window functions

Special attention should be paid to the fact that minimum attenuation of window function and that of the filter designed using that function are different in most cases. The difference, i.e. additional attenuation occurs under the process of designing a filter using window functions. This affects the stopband attenuation to become additionally higher, which is very desirable.

However, a drawback of this method is that the minimum stopband attenuation is fixed for each function. The exception is the Kaiser window described later in this chapter.

The following concepts such as the main lobe, main lobe width, side lobes, transition region, minimum stopband attenuation of window function and minimum stopband attenuation of designed filter are described in more detail in Figure 2-3-1.

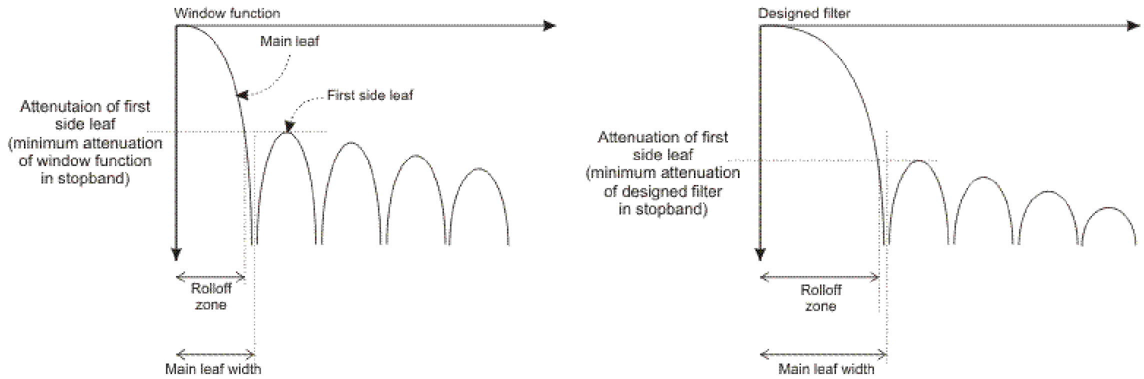


Figure 2-3-1. Main lobe, main lobe width, side lobes, transition region

As can be seen in the table 2-3-1 above, the stopband attenuation of these windows is not adjustable. It is only possible to affect the transition region by increasing the filter order. For this reason it is preferable to start design process by specifying the appropriate window function on the basis of the stopband attenuation. It is most preferable to specify a window with the least stopband attenuation that satisfies the given requirements. This enables the designed filter to have the narrowest transition region.

Example:

Design a filter with the following characteristics:

1. Minimum stopband attenuation is 40dB;
2. Transition region between 2KHz and 3KHz; and
3. Sampling frequency 10KHz.

Looking at the table 2-3-1 above (last column), it is obvious that required attenuation can be reached using the Hann window. Besides, such attenuation can be obtained using any other window following the Hann window (all the functions are sorted according to their stopband attenuation), thus the higher stopband attenuation would be at a cost of the wider transition region for the same filter order. Since the filter specification also includes transition region, this would further result in increasing the filter order and its complexity as well.

After specifying the suitable window function, it is necessary to compute the filter order. The cut-off frequencies of transition regions are normalized first.

- The sampling frequency is fs=10KHz;
- The lower cut-off frequency of transition region is f1=2KHz; and
- The upper cut-off frequency of transition region is f2=3KHz.

Normalized frequencies are obtained in the following way:

$$fn_1 = \frac{f_1}{f_s/2}\pi = \frac{2}{5}\pi = 0.4\pi$$

$$fn_2 = \frac{f2}{f_s/2}\pi = \frac{3}{5}\pi = 0.6\pi$$

Transition region of the Hann window is 0.12π for a 20th order filter. To satisfy the given specifications, the required transition region is expressed as:

$$fn2 - fn1 = 0.2\pi$$

Since the required transition region is wider than that of a 20th order filter, the filter order needs to be less than 20. It is found via iteration. Roughly estimated, the initial solution in this case can be a 12th order filter. The required order is somewhat higher and is found after performing several iterations. The resulting filter order is 16.

Filter Designer Tool is used for obtaining a first-order filter. The table 2-3-2 provides performed iterations and other important information.

NUMBER OF ITERATION	FILTER ORDER	FILTER ATTENUATION AT 3KHZ
1	12	25 dB
2	14	33 dB
3	16	49 dB
4	15	38 dB

Table 2-3-2. Calculating filter order

After specifying the window function and filter order, it is necessary to compute window coefficients w[n] using expressions for the specified window.

2.3.1 Rectangular Window

The rectangular window is rarely used for its low stopband attenuation. The first lobe (refer to Figure 2-3-2) has attenuation of 13dB and the narrowest transition region, therefore. A filter designed using this window has minimum stopband attenuation of 21 dB.

Unlike other window functions being a kind of compromise between requirements for as narrow transition region as possible and as high stopband attenuation as possible, this window is characterized by extreme values. Namely, the minimum transition region is achieved, but at a cost of stopband attenuation.

It is easy to find rectangular window coefficients as all coefficients between 0 and N-1 (N-filter order) are equal to 1, which can be expressed in the following way:

$$w[n] = 1;\ 0 \le n \le N-1$$

Note that the rectangular window performs selection of N samples from a sequence of input samples, but it does not perform sample scaling.

Figure 2-3-2 illustrates coefficients in the time-domain.

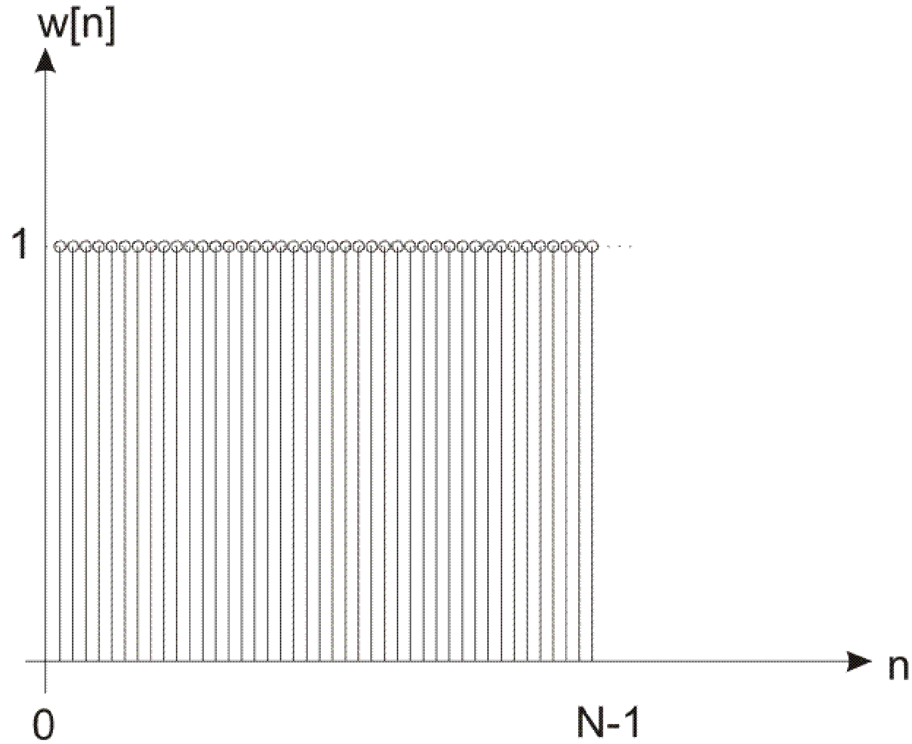


Figure 2-3-2. Rectangular window in the time domain

Figure 2-3-3 illustrates the frequency domain of rectangular window

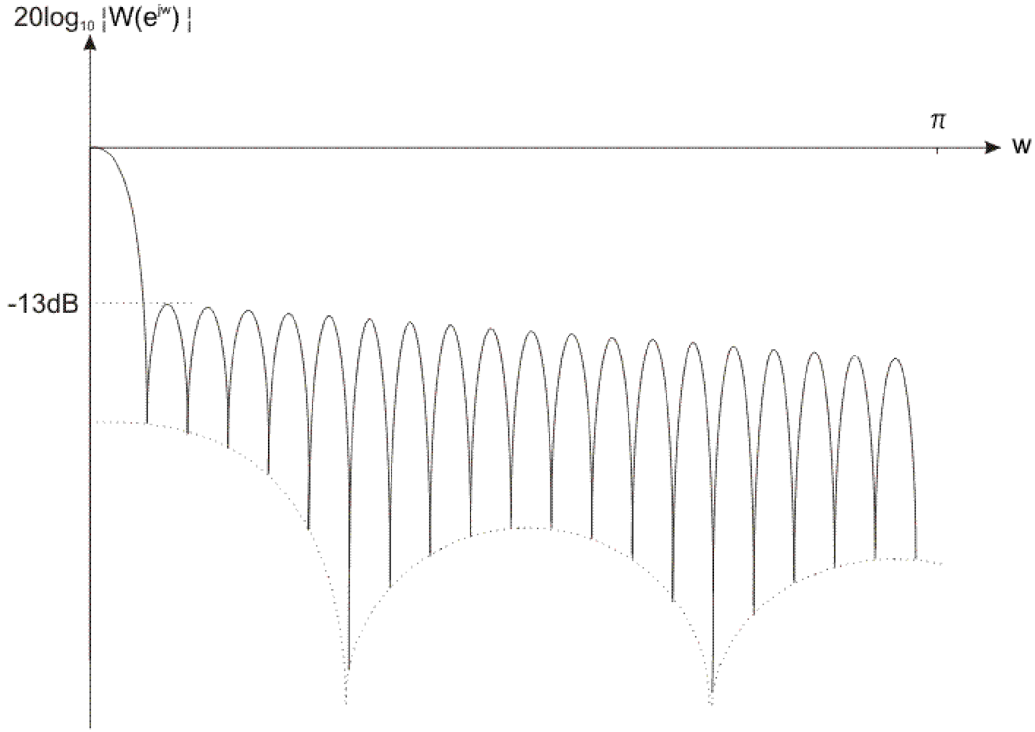


Figure 2-3-3. Rectangular window in the frequency domain (spectrum)

For its less stopband attenuation, the rectangular window is not preferable for digital filter design. Such a less attenuation is a result of cut-off samples within a window (a sequence of sampled frequencies). Up to a zero sample (from which sampling starts), all sampled frequencies are equal to zero. The first sample represents a sudden jump to some value (non-zero sample). Exactly these sudden jumps result in producing relatively sharp high-frequency components which lessen the stopband attenuation.

The attenuation gets higher by making cut-off samples less sharp, which results in reducing filter selectivity, i.e. wider transition region. Since initial requirements of a digital filter are predefined and due to less selectivity, it is necessary to increase the filter order to narrow the transition region. Note the fact that the transition region is inversely proportional to the filter order N. The transition region narrows as the filter order increases.

Increase in filter order affects the filter to become more complex and need more time for sample processing. This is why it is very important to be careful when specifying the window function and filter order as well.

2.3.2 Triangular (Bartlett) window

The triangular (Bartlett) window is one among many functions that lessens the effects of final samples. Due to it, the stopband attenuation of this window is higher than that of the rectangular window, whereas the selectivity is less. Namely, filters designed using this window have wider transition region than those designed using the rectangular window. As a result, it is necessary to have a higher order filter in order to keep the same tansition region as that of the filters designed with the rectangular window. This is the price to pay for producing the higher attenuation.

This function also represents a kind of compromise between requirements for as narrow transition region as possible and as higher stopband attenuation as possible, where the transition region is considered more important characteristic.

One of the advantages of designing filter using the triangular window is the simplicity of computing coefficients.

The triangular window coefficients can be expressed as:

$$w[n] = \begin{cases} \frac{2n}{N-1}; & 0 \le n \le \frac{N-1}{2} \\ 2 - \frac{2n}{N-1}; & \frac{N+1}{2} \le n \le N-1 \end{cases}$$

Figure 2-3-4 illustrates coefficients in the time-domain.

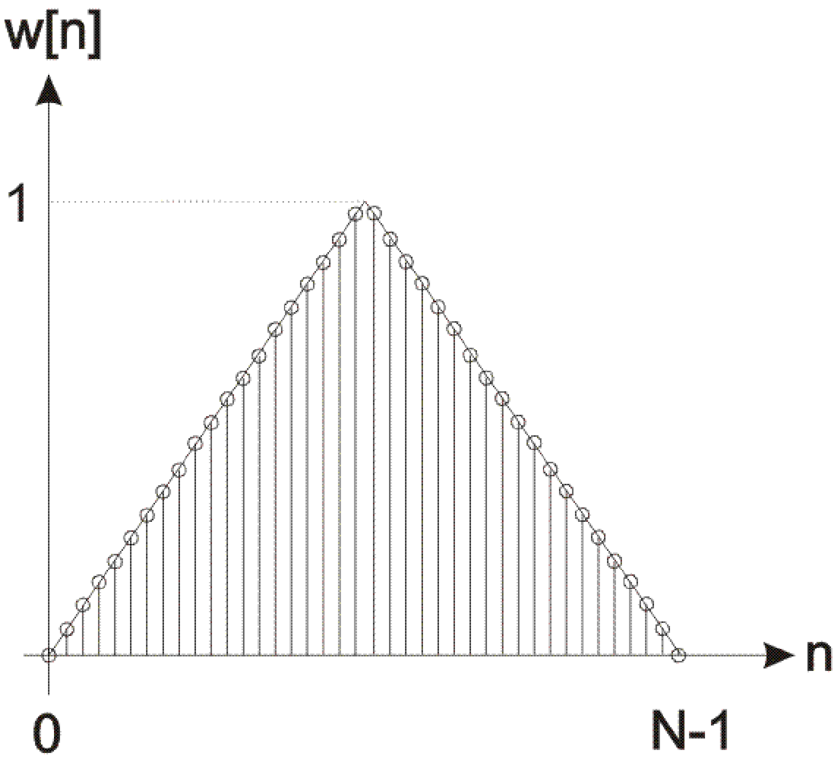


Figure 2-3-4. Triangular window coefficients in the time-domain

Figure 2-3-5. illustrates the coefficients spectrum of the triangular window shown in Figure 2-3-4.

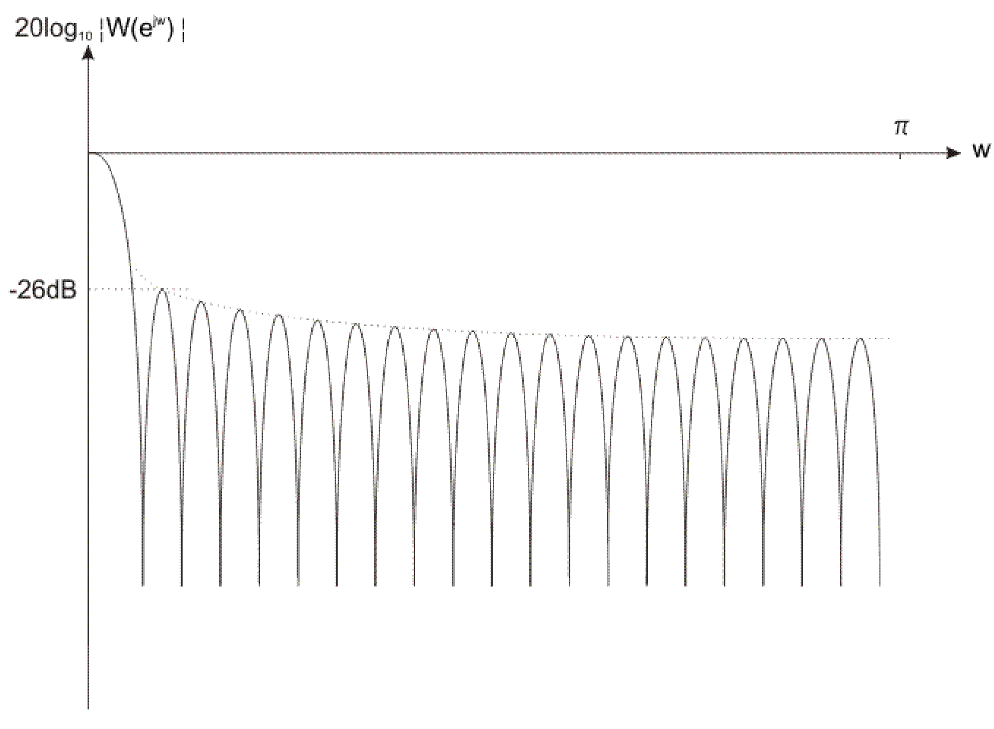


Figure 2-3-5. Triangular (Bartlett) window in the frequency domain (spectrum)

The attenuation of triangular window is low for most digital filter applications, but it is considerably higher than that for rectangular window. In some cases, when high attenuations are not needed, this filter can be used because it provides an easy way of computing coefficients.

2.3.3 Hann Window

The Hann window is used to lessen bad effects on frequency characteristic produced by the final samples of a signal being filtered. Digital filters designed with this window have higher stopband attenuation than those designed with triangle function. The first side lobe in the frequency domain of this filter has 31dB attenuation, whereas it amounts to 44dB in the designed filter. The transition region is the same as for triangular window, which makes this function one of the most desirable for designing.

Another advantage of this window is the ability to realtively fast increase the stopband attenuation of the following lobes. Already the second lobe has 41dB attenuation, whereas it amounts to 54dB for the designed filter. Refer to Figure 2-3-7 illustrating the coefficients of the Hann window.

The Hann window belongs to a class of generalized cosine windows which will be discussed in more details at the end of this chapter.

The Hann window coefficients can be expressed as:

$$w[n] = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi n}{N-1}\right)\right]; 0 \le n \le N-1$$

Figure 2-3-6. illustrates the Hann window coefficients in the time domain.

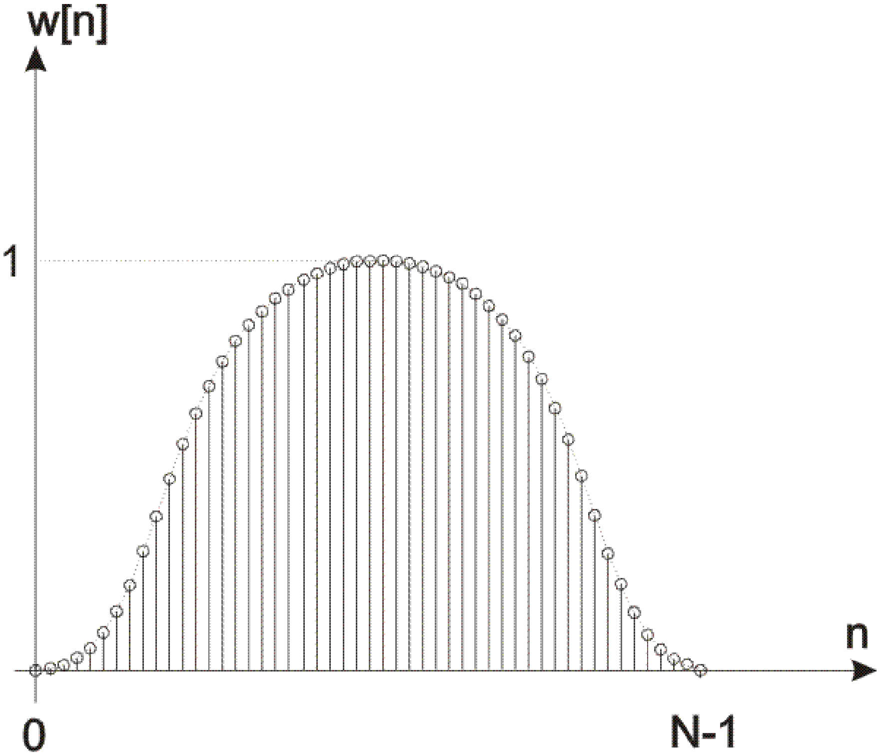


Figure 2-3-6. The Hann window coefficients in the time domain

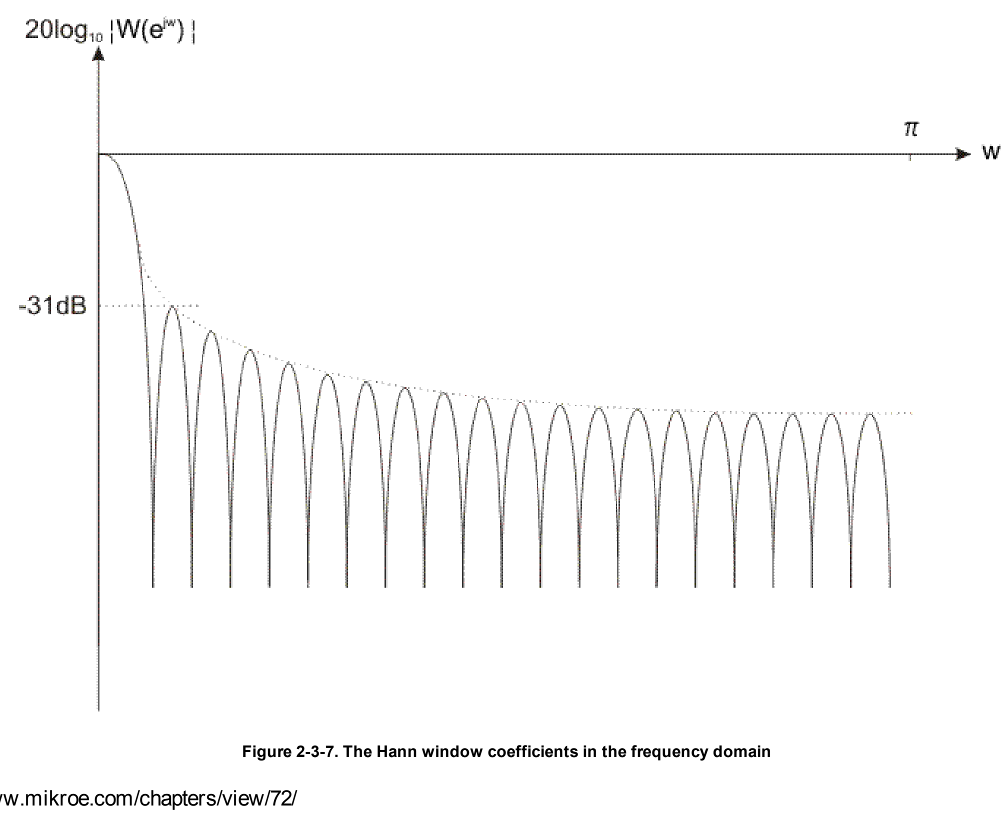


Figure 2-3-7. The Hann window coefficients in the frequency domain

As seen from the table 2-3-1, the transition region of Hann window is wider than that of triangular window. The difference is that the transition region of triangular window is computed in terms of -26dB attenuation, and in terms of -31dB attenuation for Hann window Accordingly, the conclusion is that Hann window has sharper fall than triangular one, which is considered as its advantage.

For the same requirements for minimum attenuation, the Hann window will have a narrower transition region.

2.3.4 Bartlett-Hanning Window

The Bartlett-Hannign window is another compromise between requirements for as narrow transition region as possible and as higher stopband attenuation as possible. The stopband attenuation of a filter designed with this window amounts to 39dB which is not at all a large value.

The Bartlett-Hanning window coefficients are expressed as:

$$w[n] = 0.62 - 0.48\left|\frac{n}{N-1} - 0.5\right| + 0.38\cos(2\pi(\frac{n}{N-1} - 0.5)); 0 \le n \le N-1$$

This window is in fact a combination of triangular (Bartlett) and Hann window and has higher minimum stopband attenuation than both of them. Considering that the transition region is almost the same as for triangular and Hann window, this function is one of most commonly used windows.

Figure 2-3-8. illustrates the Bartlett-Hanning window coefficients in the time domain.

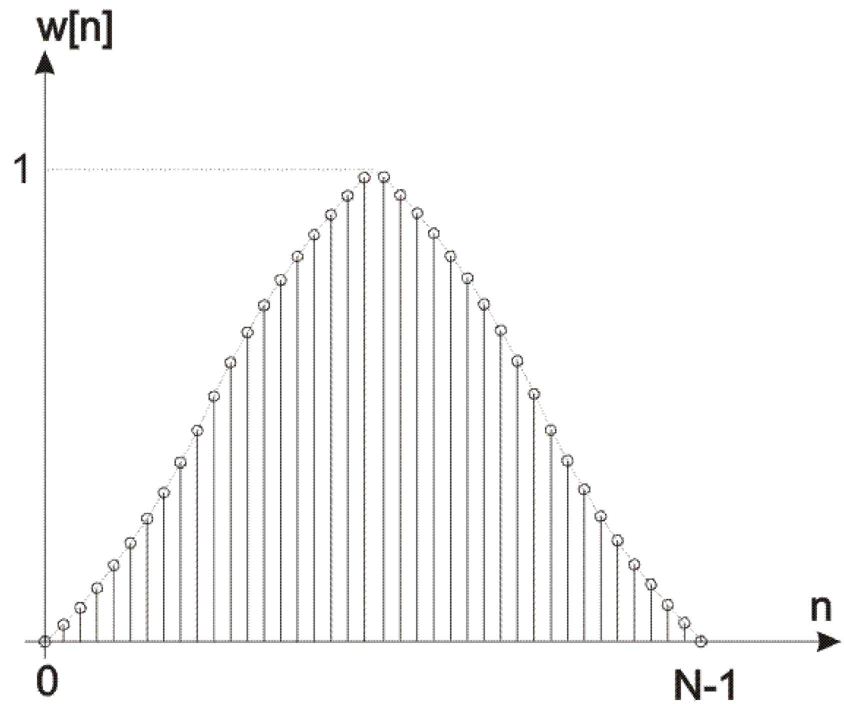


Figure 2-3-8. The Bartlett-Hanning window coefficients in the time domain

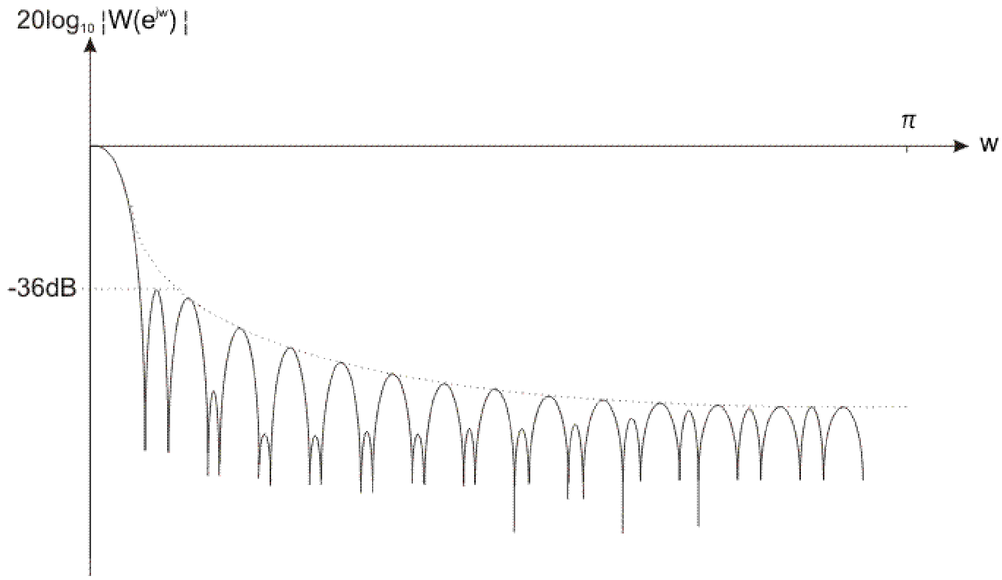


Figure 2-3-9. The Bartlett-Hanning window coefficients in the frequency domain

When the stopband attenuation doesn't need to be higher than 39dB, the Bartlett-Hanning window is one of the best solutions to use. As seen in Figure 2-3-9, the side lobes increase attenuation, which is one of very desirable characteristics of digital filters.

2.3.5 Hamming Window

The Hamming window is one of the most popular and most commonly used windows. A filter designed with the Hamming window has minimum stopband attenuation of 53dB, which is sufficient for most implementations of digital filters. The transition region is somewhat wider than that of the Hann and Bartlett-Hanning windows, whereas the stopband attenuation is considerably higher. Unlike minimum stopband attenuation, the transition region can be changed by changing the filter order. The transition region narrows, whereas the minimum stopband attenuation remains unchanged as the filter order increases.

The Hamming window coefficients are expressed as:

$$w[n] = 0.54 - 0.46(1 - \cos(\frac{2\pi n}{N-1})); 0 \le n \le N-1$$

The Hamming window belongs to a class of generalized cosine functions which are described in more details at the end of this chapter.

Figure 2-3-10 illustrates the Hamming window coefficients in the time domain.

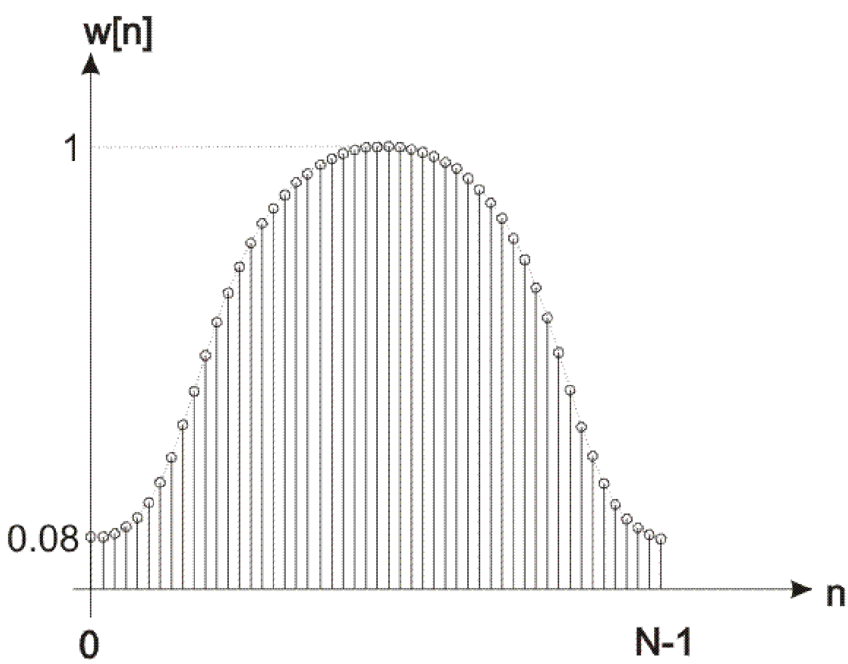


Figure 2-3-10. The Hamming window coefficients in the time domain

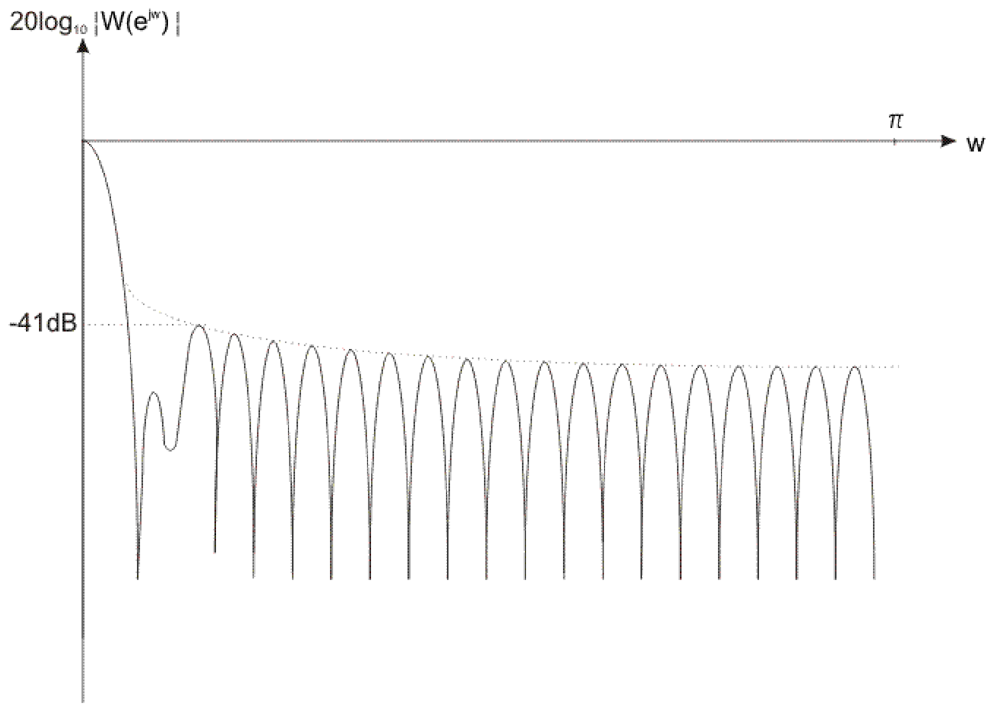


Figure 2-3-11. The Hamming window coefficients in the frequency domain

Figure 2-3-11 illustrates the Hamming window in the frequency domain. As seen, the first side lobe is attenuated, so that the minimum stopband attenuation is defined in terms of the second side lobe and amounts to 41dB. All side lobes have almost the same maximum values (about -45dB).

2.3.6 Bohman Window

The Bohman window is a convolution of two semi-periods of a cosine function. The transition region and the main lobe are wider than those for Hamming window, but the stopband attenuation is higher, therefore. The attenuation of the first side lobe for Bohman window is 46dB, whereas the filters designed with Bohman window have the stopband attenuation of 51dB.

The Bohman window coefficients are expressed as:

$$w[n]=\left(1-\frac{\left|n-\frac{N-1}{2}\right|}{\frac{N-1}{2}}\right)\cos\left(\pi\frac{\left|n-\frac{N-1}{2}\right|}{\frac{N-1}{2}}\right)+\frac{1}{\pi}\sin\left(\pi\frac{\left|n-\frac{N-1}{2}\right|}{\frac{N-1}{2}}\right);0\le n\le N-1$$

Figure 2-3-12 illustrates the Bohman window coefficients in the time domain, whereas Figure 2-3-13 illustrates its coefficients in the frequency domain.

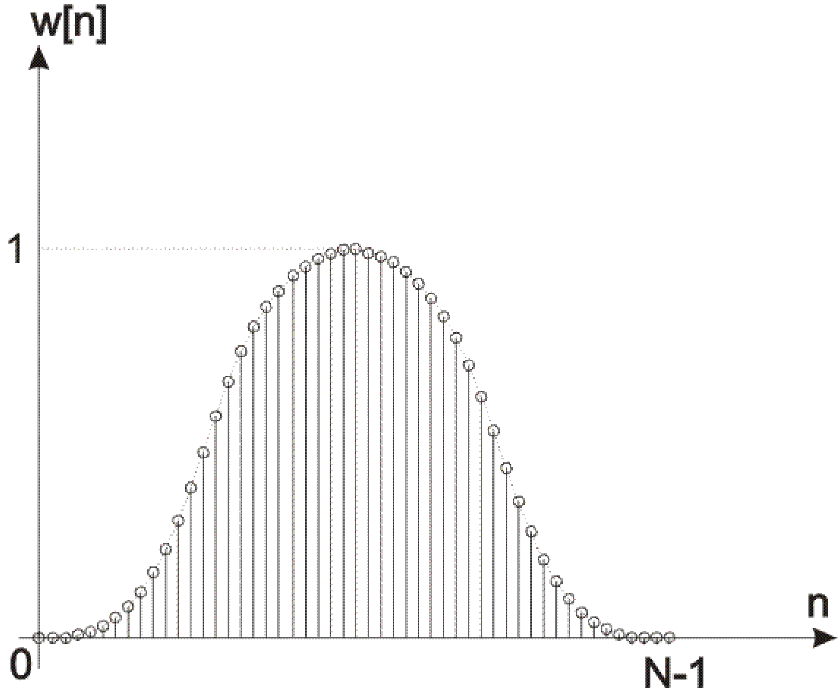


Figure 2-3-12. The Bohman window coefficients in the time domain

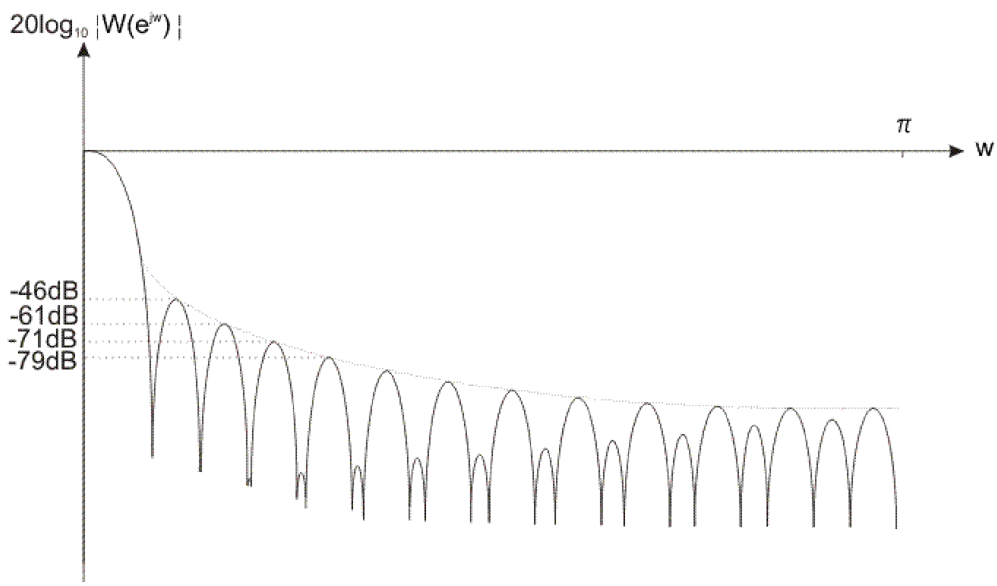


Figure 2-3-13. The Bohman window coefficients in the frequency domain

Figure 2-3-13 illustrates the Bohman window coefficients in the frequency domain. As seen, the first side lobe has minimum attenuation of 46dB, whereas for designed filter it amounts to 51dB. The attenuation increases relatively fast. The second side lobe in the Bohman window frequency domain has the attenuation of 61dB, whereas for the filters designed with this window it amounts to 65dB. It should be noted that side lobes of such filters are different than shown in Figure 2-3-13.

With regard to minimum attenuation as well as to mid-wide transition region, it can be concluded that Bohman window is suitable for most applications.

2.3.7 Blackman Window

The Blackman window is, along with Kaiser, Hamming and Blackman-Harris windows, considered most commonly used and the most popular windows. Relatively high attenuation makes this window very convenient for almost all applications. The minimum stopband attenuation of a filter designed with this window amounts to 75dB.

The Blackman window coefficients are expressed as:

$$w[n] = 0.42 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right); 0 \le n \le N-1$$

Figure 2-3-14 illustrates the Blackman window coefficients in the time domain, whereas Figure 2-3-15. illustrates its coefficients in the frequency domain.

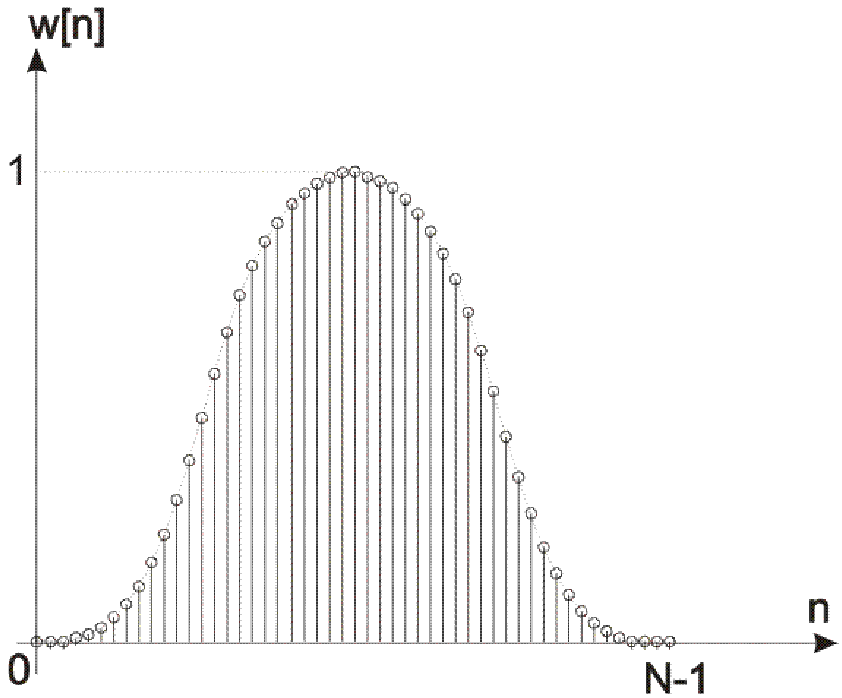


Figure 2-3-14. The Blackman window coefficients in the time domain

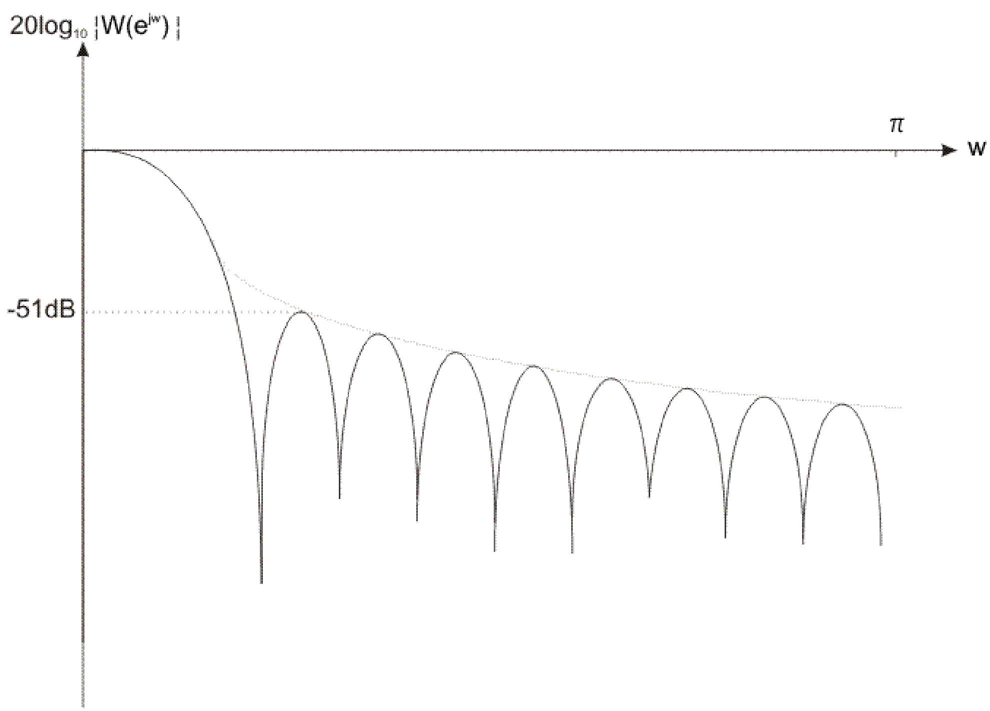


Figure 2-3-15. The Blackman window coefficients in the frequency domain

As seen from Figure 2-3-15, the Blackman window frequency domain reminds of the Hann window frequency domain. The difference is in attenuation of the first side lobe which amounts to 51dB, as well as in the main lobe which is somewhat wider. The side lobes, following the first one, cause additional stopband attenuation.

2.3.8 Blackman-Harris Window

The Blackman-Harris window is definately one of the most well-known and most commonly used windows. It is characterized by high stopband attenuation and the widest transition region comparing to all windows mentioned in this chapter. However, increase in filter order (more complex filter) cannot affect the minimum stopband attenuation, but affects the transition region.

The Blackman-Harris window has almost twice as wide transition region than, say, the Hamming window. Such a drawback (wide transition region) can be overcome by increasing the filter order. The result is a higher order filter (comparing to one designed with Hamming window) with twice as high stopband attenuation. It should be noted that the transition region of the Hamming window ranges between − 3dB and − 41dB (decrease of 38dB), whereas for Blackman-Harris window it is decreased by 88dB (from − 3dB to − 91dB).

The Blackman-Harris window has minimum stopband attenuation of 91dB, whereas for the filters designed with this window it amounts to 109dB. Such an attenuation is sufficient for any digital filter application. If it is possible to support a filter order that will be required by this window, then the Blackman-Harris window is probably the best soultion.

The Blackman-Harris window coefficients are expressed as:

$$w[n] = 0{,}35875 - 0{,}48829\cos\left(\frac{2\pi n}{N-1}\right) + 0{,}14128\cos\left(\frac{4\pi n}{N-1}\right) - 0{,}01168\cos\left(\frac{6\pi n}{N-1}\right); 0 \le n \le N-1$$

Figure 2-3-16. illustrates the Blackman-Harris window coefficients in the time domain, whereas Figure 2-3-16. illustrates its coefficients in the frequency domain.

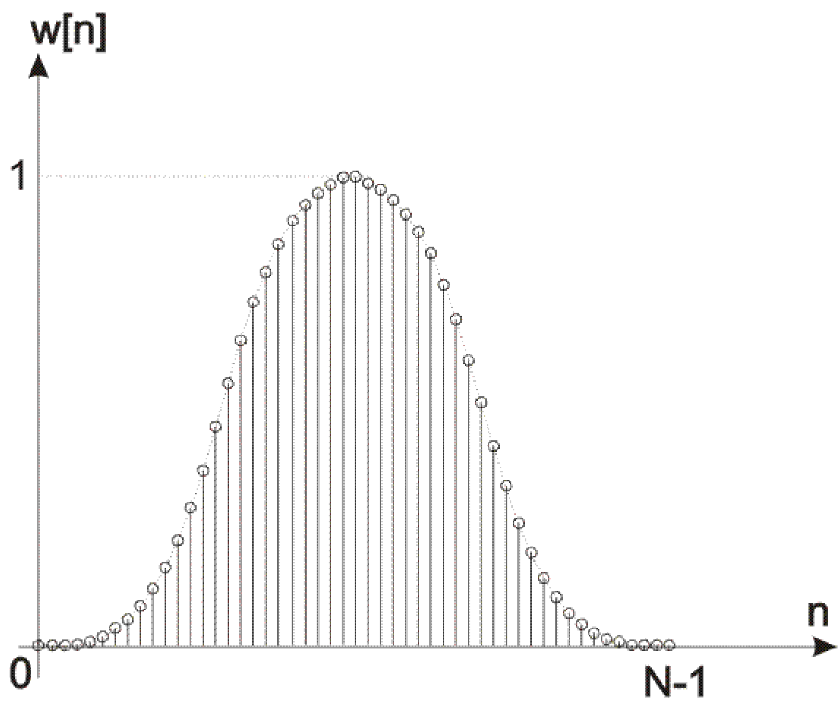


Figure 2-3-16. The Blackman-Harris window coefficients in the time domain

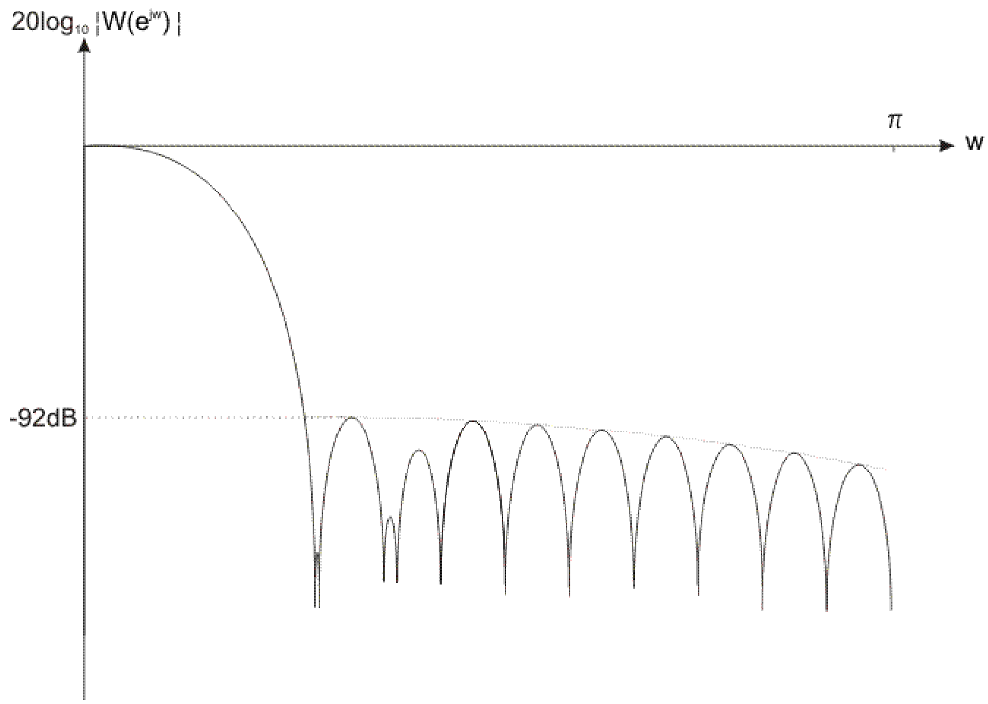


Figure 2-3-17. The Blackman-Harris window coefficients in the frequency domain

Figure 2-3-17. illustrates the main characteristics of the Blackman-Harris window in the frequency domain. As seen, the main lobe is wide and the first side lobe is suppressed. Minimum stopband attenuation is determined by the second side lobe having the minimum attenuation of 91dB. The attenuation of other side lobes gradually increases.

2.3.9 Blackman-Harris Window

The generalized cosine functions are a type of window functions whose coefficients can be obtained via expression:

$$w[n] = A - B \cdot \cos\left(\frac{2\pi n}{N-1}\right) + C \cdot \cos\left(\frac{4\pi n}{N-1}\right); 0 \le n \le N-1$$

There are some window functions that also belong to this class. These are:

- Rectangular window;
- Hann window;
- Hamming window; and
- Blackman window.

Each of these window functions is only a variation of the expression above. Their coefficients a, b and c are given in the table 2-3-3 below.

WINDOW FUNCTION	A	B	C
Rectangular	1	0	0
Hann	0.5	0.5	0
Hamming	0.54	0.46	0
Blackman	0.42	0.5	0.08

Table 2-3-3. The coefficients of generalized cosine window functions

2.3.10 Kaiser Window

As you know, each of the windows described above is a kind of compromise between requirements for as narrow transition region as possible (greater selectivity) and as higher stopband attenuation as possible.

Comparing Hann and Bartlett-Hanning windows, it is obvious that both of them have the same transition region, but the Bartlett-Hanning window has higher attenuation. There is one more thing of concern which says that the minimum stopband attenuation depends on the specified window, whereas an increase in filter order affects the transition region.

All this leads us to the conclusion that the windows described here are not optimal. An optimal window is a function that has maximum attenuation according to the given width of the main lobe. The optimal window is also known as Kaiser window.

Its coefficients are expressed as:

$$w[n] = \frac{I_0\left(\beta \cdot \sqrt{1-\left(\frac{n-\alpha}{\alpha}\right)^2}\right)}{I_0(\beta)}; 0 \le n \le N-1$$

$$\alpha = \frac{N-1}{2}$$

$$N = \frac{a_a - 8}{4.57 \cdot \Delta\omega} + 1$$

where aa is the minimum stopband attenuation, and Δ ω is the width of (normalized) transition region. The order of band-pass and band-stop filters, obtained from the expression above, should be multiplied by 2.

The value of parameter β can be obtained from the table 2-3-4.

AA	B
less than 21	0
between 21 and 50	0.5842(aa - 21)^0.4 + 0.07886(aa - 21)
more than 50	0.1102(aa - 8.7)

Table 2-3-4. Values of parameter β

I0(*) is a modified zero order Bessel function of the first kind. It can be approximated via expression:

$$I_0(x) \cong 1 + \sum_{k=1}^{20} \left(\frac{\left(\frac{x}{2}\right)^k}{k!} \right)^2$$

In most practical cases it is sufficient to consider the first 20 elements of this order.

As can be seen from all mentioned above, in order to design an optimal Kaiser filter it is necessary to know normalized width of transition region as well as minimum desirable stopband attenuation.

Example:

What would a filter design with Kaiser window look like if we take into consideration the requirements given at the beginning of this chapter? The result of a filter designed with the Hann window is a 16th order filter.

Assuming that it is required to design a filter with the following characteristics:

1. Minimum stopband attenuation is 40dB (aa);
2. Transition region is between 2KHz and 3KHz (f1, f2); and
3. Sampling frequency is 10KHz (fs).

Transition region:

$$f_p = \pi \frac{f_1}{f_s/2} = \pi \frac{2}{5} = 0.4\pi$$

$$f_s = \pi \frac{f_2}{f_2/2} = \pi \frac{3}{5} = 0.6\pi$$

$$\Delta\omega = f_s - f_p = 0.2\pi$$

$$N = \frac{a_a - 8}{4.57 \cdot \Delta\omega} + 1 = \frac{40 - 8}{4.57 \cdot 0.2\pi} + 1 = 12.15$$

The specified filter order is N=12. It means that the needed filter order is less by 4 than that obtained with the Hann window.

It is important to say that all the expressions above are obtained in an empirical way, which means that there will be some exceptions and variations in practice. For example, the expression used to compute the filter order gives accurate results in approximately 98% cases. Otherwise, the resulting filter order should be changed. Fortunately, the changes to be made are slight, and the filter order is increased or decreased by 2 at most.

2.4 Examples

This chapter discusses various FIR filter design methods. It also provides examples of all types of filters as well as of all methodes described in the previous chapters. The four standard types of filters are used here:

- low-pass filter;
- high-pass filter;
- band-pass filter; and
- band-stop filter.

The design method used here is known as the window method.

The FIR filter design process can be split into several steps as described in Chapter 2.2.4 entitled *Designing FIR filters using window functions*. These are:

1. Defining filter specifications;
2. Specifying a window function according to the filter specifications;
3. Computing the filter order according to the filter specifications and specified window function;
4. Computing the coefficients of the window;
5. Computing the ideal filter coefficients according to the filter order;
6. Computing the FIR filter coefficients according to the obtained window function and ideal filter coefficients; and
7. If the resulting filter has too wide or too narrow transition region, it is necessary to change the filter order. The specified filter order is increased or decreased according to needs, and steps 4, 5 and 6 are repeated after that as many times as needed.

Depending on the window function in use, some steps will be skipped. If the filter order is known, step 3 is skipped. If the window function to use is predetermined, step 2 is skipped.

In every given example, the FIR filter design process will be described through these steps in order to make it easier for you to note similarities and differencies between various design methodes, window functions and design of various types of filters as well.

2.4.1 Filter design using Rectangular window

2.4.1.1 Example 1

Step 1:

Type of filter – low-pass filter
Filter specifications:

- Filter order – N=10
- Sampling frequency – fs=20KHz
- Passband cut-off frequency – fc=2.5KHz

Step 2:

Method – filter design using rectangular window

Step 3:

Filter order is predetermined, N=10;
A total number of filter coefficients is larger by one, i.e. N+1=11; and
Coefficients have indices between 0 and 10.

Step 4:

All coefficients of the rectangular window have the same value equal to 1.

$$w[n] = 1 \; ; \; 0 \le n \le 10$$

Step 5:

The ideal low-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \dfrac{\sin[\omega_c(n-M)]}{\pi(n-M)}; & n \ne M \\ \dfrac{\omega_c}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N}{2} = 5$$

Normalized cut-off frequency ωc can be calculated using the following expression:

$$\omega_c = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 2500}{20000} = 0.25\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc with expression for the impulse response coefficients of the ideal low-pass filter:

$$h_d[n]=\{-0.045016; 0; 0.075026; 0.159155; 0.225079; \\ 0.25; \\ 0.225079; 0.159155; 0.075026; 0; -0.045016\}$$

The middle element is found via the following expression $\frac{\omega_c}{\pi}$

Step 6:

The designed FIR filter coefficients are obtained via the following expression:

$$h[n]=w[n] \cdot h_d[n] \; ; \; 0 \le n \le 10$$

The FIR filter coefficients h[n] rounded to 6 digits are:

$$h[n]=\{-0.045016; 0; 0.075026; 0.159155; 0.225079; \\ 0.25; \\ 0.225079; 0.159155; 0.075026; 0; -0.045016\}$$

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-1 illustrates the direct realization of designed FIR filter, whereas Figure 2-4-2 illustrates the optimized realization of designed FIR filter, which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

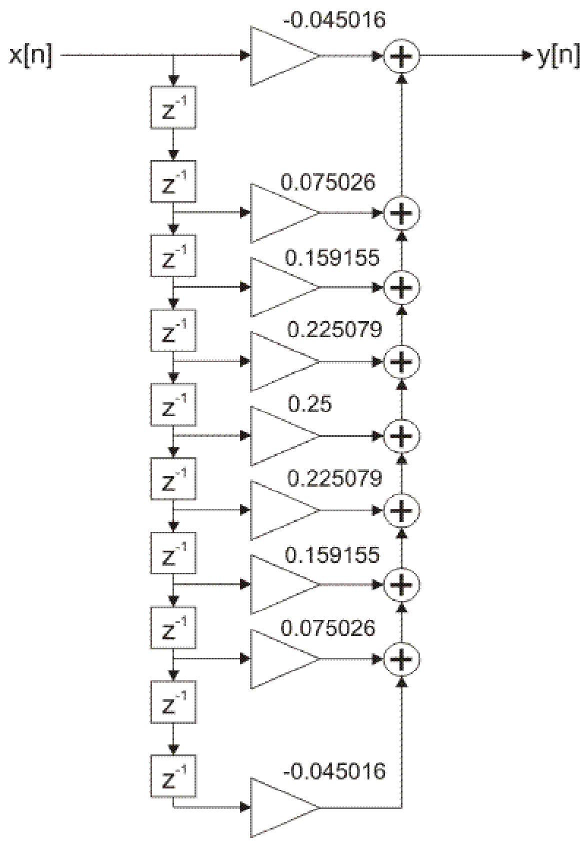


Figure 2-4-1. FIR filter direct realization

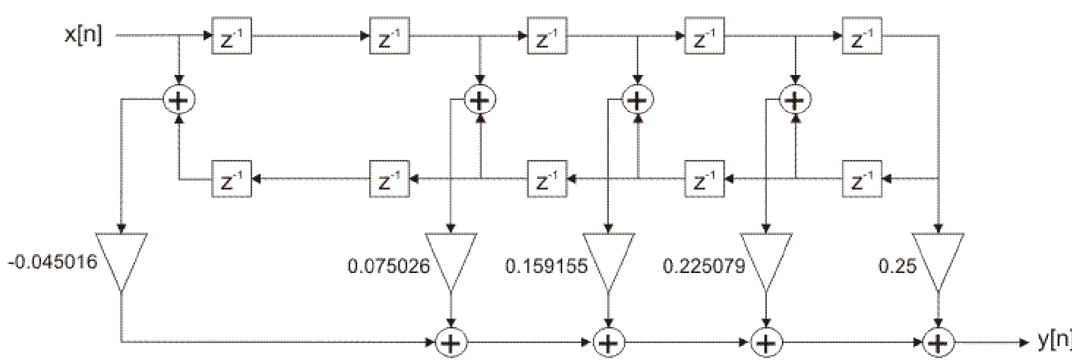


Figure 2-4-2. Optimized realization structure of FIR filter

2.4.1.2 Example 2

Step 1:

Type of filter – high-pass filter
Filter specifications:

- Filter order – N=8
- Sampling frequency– fs=20KHz
- Passband cut-off frequency – fc=5KHz

Step 2:

Method – filter design using rectangular window

Step 3:

Filter order is predetermined, N=8;
A total number of filter coefficients is larger by 1, i.e. N+1=9;
Coefficients have indices between 0 and 8.

Step 4:

All coefficients of the rectangular window have the same value equal to 1.

w[n] = 1 ; 0 ≤ n ≤ 8

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} 1 - \frac{\omega_c}{\pi}; & n \neq M \\ -\frac{\sin(\omega_c(n-M))}{\pi(n-M)}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N}{2} = 4$$

Normalized cut-off frequency ωc can be calculated using the following expression:

$$\omega_c = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 5000}{20000} = 0.5\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc with expression for the impulse response coefficients of the ideal low-pass filter:

$$h_d[n] = \{ 0, 0.106103, 0, -0.318310, \\ 0.5, \\ -0.318310, 0, 0.106103, 0\}$$

Step 6:

The designed FIR filter coefficients are found via expression:

$$h[n] = w[n] \cdot h_d[n] \quad ; 0 \leq n \leq 8$$

The FIR filter coefficients h[n] rounded to 6 digits are:

$$h[n] = \{ 0, 0.106103, 0, -0.31831, \\ 0.5, \\ -0.31831, 0, 0.106103, 0\}$$

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-3 illustrates the direct realization of designed FIR filter, whereas figure 2-4-4 illustrates the optimized realization of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

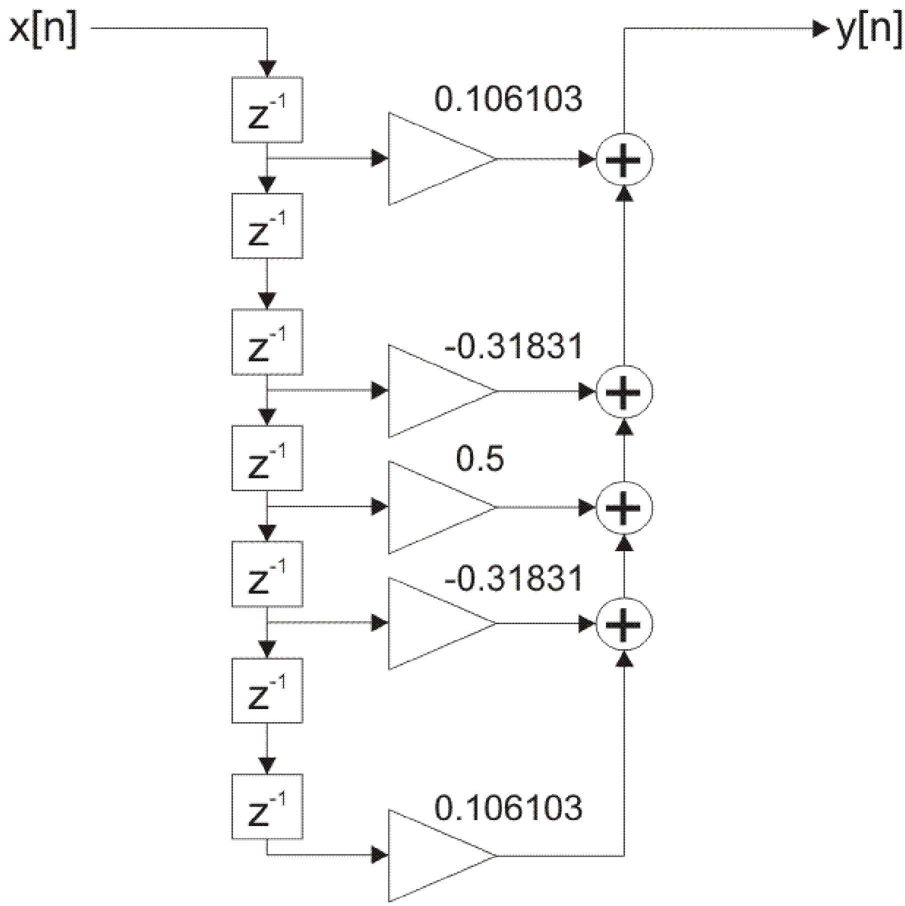


Figure 2-4-3. FIR filter direct realization

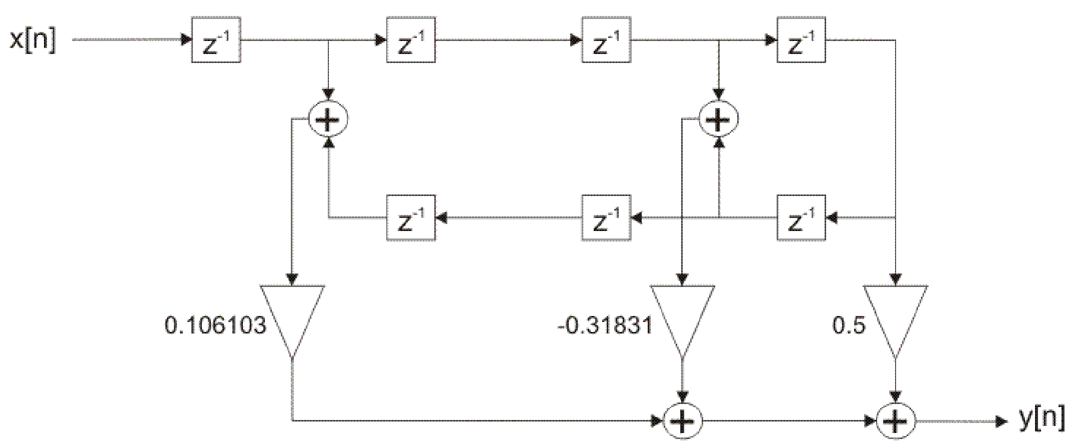


Figure 2-4-4. FIR filter optimized realization structure

2.4.1.3 Example 3

Step 1:

Type of filter – band-pass filter
Filter specifications:

- Filter order – N=14
- Sampling frequency – fs=20KHz
- Passband cut-off frequency – fc1=3KHz, fc2=5.5KHz

Step 2:

Method – filter design using rectangular window

Step 3:

Filter order is predetermined, N=14

A total number of filter coefficients is larger by 1, i.e. N+1=15.
Coefficients have indices between 0 and 14.

Step 4:

All coefficients of the rectangular window have the same value equal to 1.

$$w[n] = 1 \; ; \; 0 \le n \le 14$$

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \dfrac{\sin(\omega_{c2}(n-M))}{\pi(n-M)} - \dfrac{\sin(\omega_{c1}(n-M))}{\pi(n-M)}; & n \neq M \\ \dfrac{\omega_{c2} - \omega_{c1}}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N}{2} = 7$$

Normalized cut-off frequencies ωc1 and ωc2 can be found using the following expressions:

$$\omega_{c1} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 3000}{20000} = 0.3\pi$$

$$\omega_{c2} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 5500}{20000} = 0.55\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc1 and ωc2 with expression for the impulse response coefficients of the ideal band-pass filter:

h[n]= { -0.034696, -0.011737, 0.108678, 0.093549, -0.127326, -0.200547, 0.056873,
0.25,
0.056873, -0.200547, -0.127326, 0.093549, 0.108678, -0.011737, -0.034696 }

Step 7:

Filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-5 illustrates the direct realization of designed FIR filter, whereas figure 2-4-6 illustrates optimized realization of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

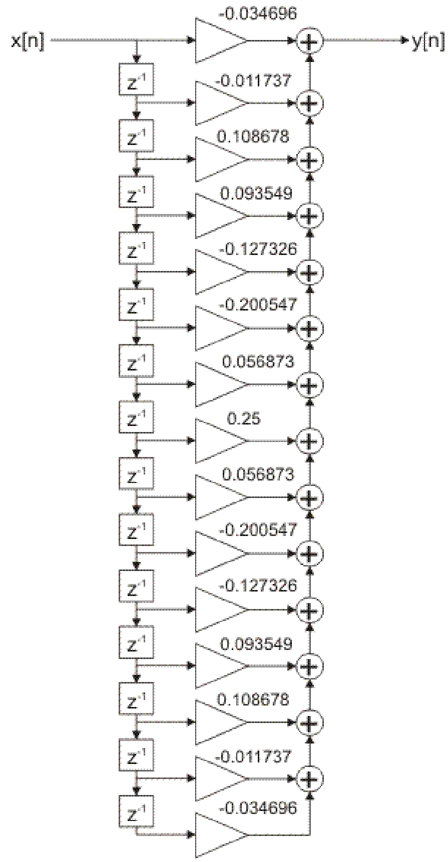


Figure 2-4-5. FIR filter direct realization

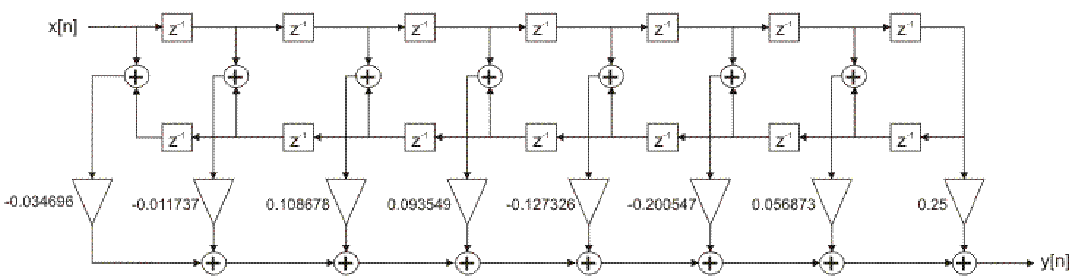


Figure 2-4-6. FIR filter optimized realization structure

2.4.1.4 Example 4

Step 1:

Type of filter – band-stop filter
Filter specifications:

- Filter order – N=14
- Sampling frequency – fs=20KHz
- Stopband cut-off frequency - fc1=3KHz, fc2=5.5KHz

Step 2:

Method – filter design using rectangular window

Step 3:

Filter order is predetermined, N=14;
A total number of filter coefficients is larger by 1, i.e. N+1=15; and
Coefficients have indices between 0 and 14.

Step 4:

All coefficients of the rectangular window have the same value equal to 1.

$$w[n] = 1 \ ; \ 0 \le n \le 14$$

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \dfrac{\sin(\omega_{c1}(n-M))}{\pi(n-M)} - \dfrac{\sin(\omega_{c2}(n-M))}{\pi(n-M)}; & n \neq M \\ 1 - \dfrac{\omega_{c2} - \omega_{c1}}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N}{2} = 7$$

Normalized cut-off frequencies ωc1 and ωc2 can be found using expressions:

$$\omega_{c1} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 3000}{20000} = 0.3\pi$$

$$\omega_{c2} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 5500}{20000} = 0.55\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc1 and ωc2 with expression for the impulse response coefficients of the ideal band-stop filter:

$h_d[n]$={ 0.034696, 0.011737, -0.108678, -0.093549, 0.127326, 0.200547, -0.056873,
0.75,
-0.056873, 0.200547, 0.127326, -0.093549, -0.108678, 0.011737, 0.034696 }

Note that, excepting the middle element, all coefficients are the same as in the previous example (band-pass filter with the same cut-off frequencies), but have the opposite sign.

Step 6:

The designed FIR filter coefficients are found via expression:

$$h[n]=w[n]\cdot h_d[n] \quad ; 0\leq n\leq 14$$

The FIR filter coefficients h[n], rounded to 6 digits, are:

h[n]= { 0.034696, 0.011737, -0.108678, -0.093549, 0.127326, 0.200547, -0.056873,
0.75,
-0.056873, 0.200547, 0.127326, -0.093549, -0.108678, 0.011737, 0.034696 }

Step 7:

Filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-7 illustrates the direct realization of designed FIR filter, whereas figure 2-4-8 illustrates optimized realization of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

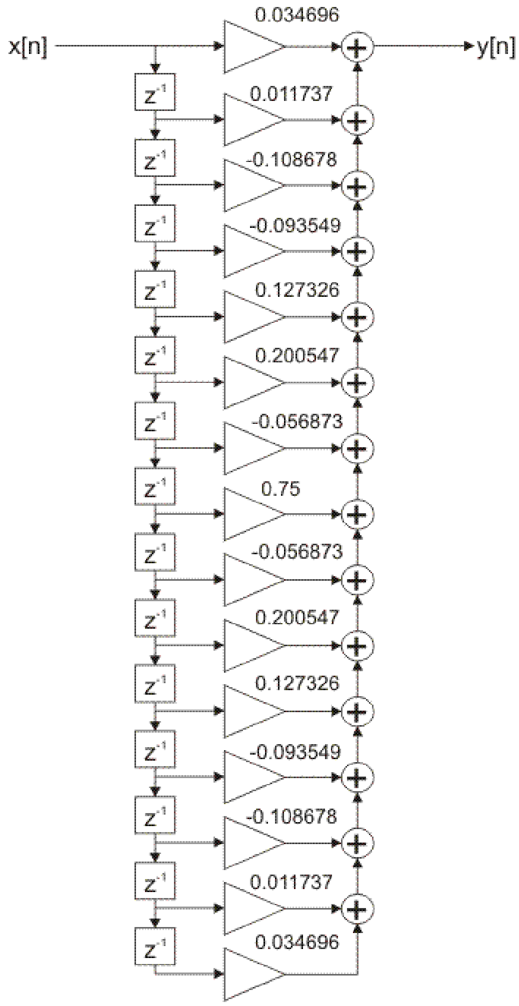


Figure 2-4-7. FIR filter direct realization

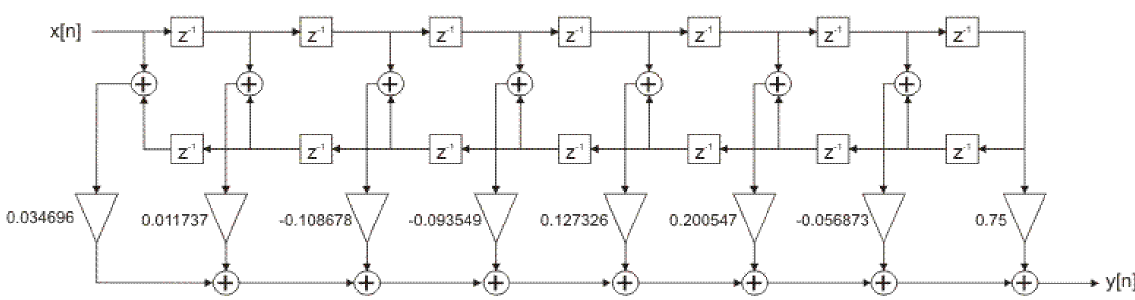


Figure 2-4-8. FIR filter optimized realization structure

2.4.2 Filter design using Bartlett window

2.4.2.1 Example 1

Step 1:

Type of filter – low-pass filter
Filter specifications:

- Filter order– Nf=9
- Sampling frequency – fs=20KHz
- Passband cut-off frequency – fc=2.5KHz

Step 2:

Method – filter design using Barlett window

Step 3:

Filter order is predetermined, Nf=9;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=10; and

Coefficients have indices between 0 and 8.

Step 4:

The coefficients of Bartlett window are expressed as:

$$w[n] = \begin{cases} \frac{2n}{N-1}; & 0 \le n \le \frac{N-1}{2} \\ 2 - \frac{2n}{N-1}; & \frac{N+1}{2} \le n \le N-1 \end{cases}$$

$$w[n]=\{ 0, 0.222222, 0.444444, 0.666667, 0.888889, 0.888889, 0.666667, 0.444444, 0.222222, 0 \}$$

Step 5:

The ideal low-pass filter coefficients (ideal filter impulse response) are given in the expression below:

$$h_d[n] = \begin{cases} \frac{\sin[\omega_c(n-M)]}{\pi(n-M)}; & n \neq M \\ \frac{\omega_c}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N_f}{2} = 4.5$$

Since the value of M is not an integer, the middle element representing a center of coefficients symmetry doesn't exist.

Normalized cut-off frequency ωc can be calculated using expression:

$$\omega_c = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 2500}{20000} = 0.25\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc with expression for the impulse response coefficients of the ideal low-pass filter:

$$h_d[n]=\{ -0.027069, 0.034803, 0.117632, 0.196053, 0.243624, 0.243624, 0.196053, 0.117632, 0.034803, -0.027069\}$$

Step 6:

The designed FIR filter coefficients are found via expression:

$$h[n] = w[n] * hd[n] ; 0 \le n \le 9$$

The FIR filter coefficients h[n] rounded to 6 digits are:

$$h[n]=\{ 0, 0.007734, 0.052281, 0.130702, 0.216555, 0.216555, 0.130702, 0.052281, 0.007734, 0\}$$

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-9 illustrates the direct realization of designed FIR filter, whereas figure 2-4-10 illustrates optimized realization of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

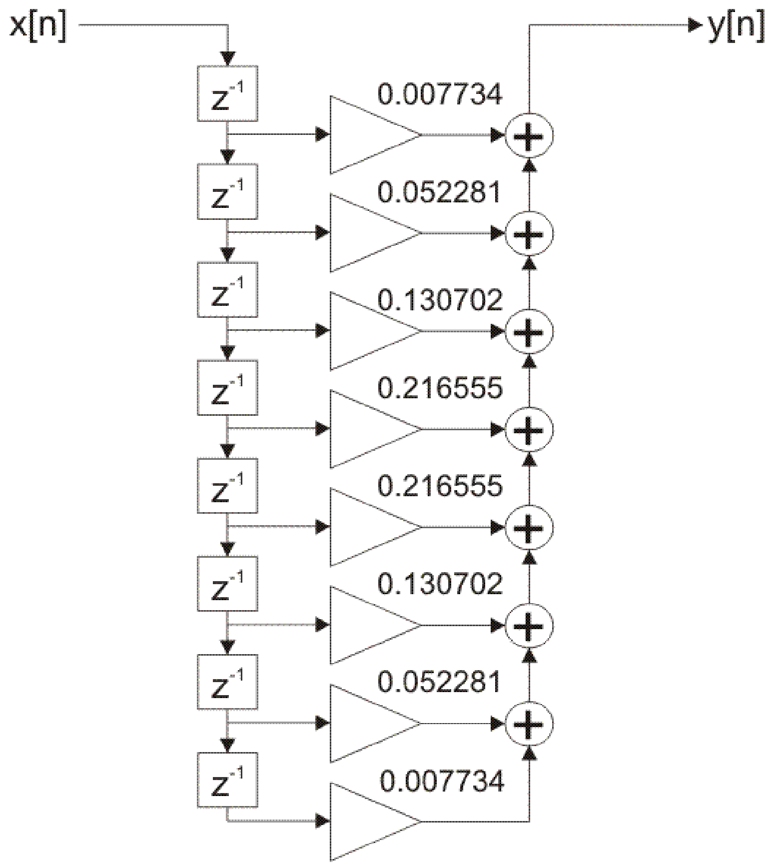


Figure 2-4-9. FIR filter direct realization

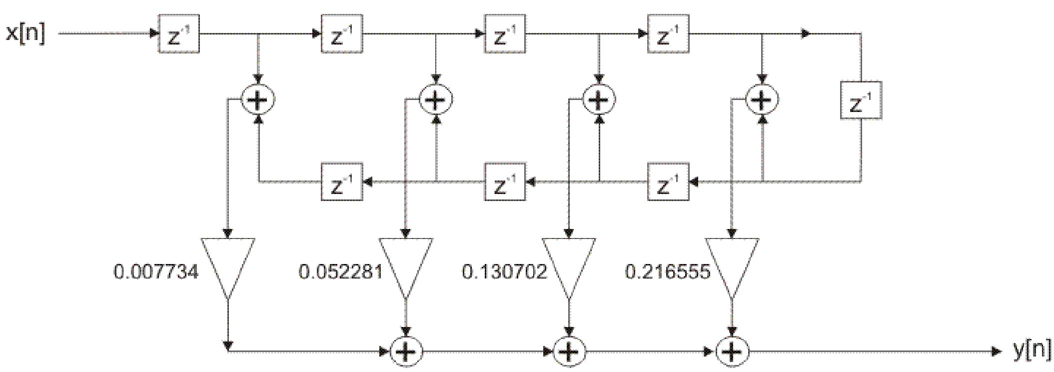


Figure 2-4-10. FIR filter optimized realization structure

2.4.2.2 Example 2

Step 1:

Type of filter – high-pass filter
Filter specifications:

- Filter order – Nf=8
- Sampling frequency – fs=20KHz
- Passband cut-off frequency – fc=5KHz

Step 2:

Method –filter design using Bartlett window

Step 3:

Filter order is predetermined, Nf=8;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=9;
Coefficients have indices between 0 and 8.

Step 4:

The Bartlett window function coefficients are found via expression:

$$w[n] = \begin{cases} \dfrac{2n}{N-1}; & 0 \le n \le \dfrac{N-1}{2} \\ 2 - \dfrac{2n}{N-1}; & \dfrac{N+1}{2} \le n \le N-1 \end{cases}$$

$$w[n]=\{\ 0,\ 0.25,\ 0.5,\ 0.75,\ 1,\ 0.75,\ 0.5,\ 0.25,\ 0\ \}$$

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} 1 - \dfrac{\omega_c}{\pi}; & n \ne M \\ -\dfrac{\sin(\omega_c(n-M))}{\pi(n-M)}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N}{2} = 4$$

Normalized cut-off frequency ωc may be calculated via the following expression:

$$\omega_c = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 5000}{20000} = 0.5\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc with expression for the impulse response coefficients of the ideal high-pass filter:

$$h_d[n]=\{\ 0,\ 0.106103,\ 0,\ -0.318310,\ 0.5,\ -0.318310,\ 0,\ 0.106103,\ 0\}$$

Step 6:

The designed FIR filter coefficients are found via expression:

$$h[n] = w[n] * hd[n]\ ;\ 0 \le n \le 8$$

The FIR filter coefficients h[n] rounded to 6 digits are:

$$h[n]=\{\ 0,\ 0.026526,\ 0,\ -0.238732,\ 0.5,\ -0.238732,\ 0,\ 0.026526,\ 0\ \}$$

Step 7:

Filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-11 illustrates the direct realization of designed FIR filter, whereas figure 2-4-12 illustrates optimized realization of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

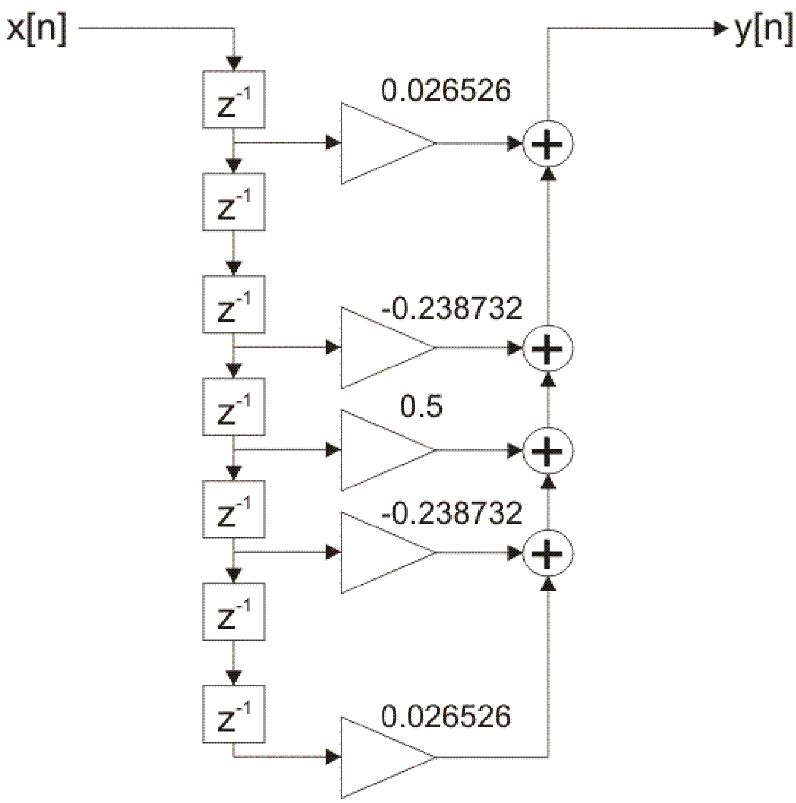


Figure 2-4-11. FIR filter direct realization

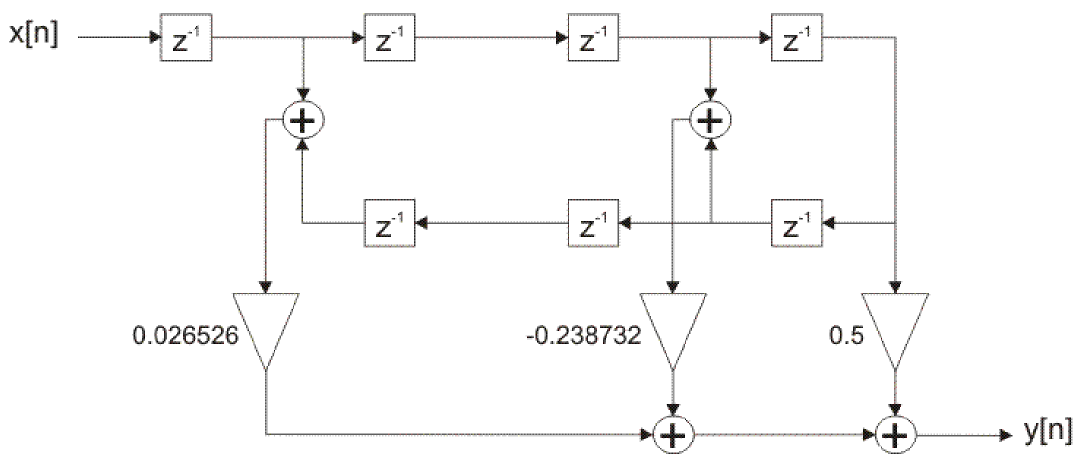


Figure 2-4-12. FIR filter optimized realization structure

2.4.2.3 Example 3

Step 1:

Type of filter – band-pass filter
Filter specifications:

- Filter order– Nf=14;
- Sampling frequency – fs=20KHz; and
- Passband cut-off frequencies – fc1=3KHz, fc2=5.5KHz.

Step 2:

Method – filter design using Bartlett window

Step 3:

Filter order is predetermined, Nf=14;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=15; and
Coefficients have indices between 0 and 14.

Step 4:

The Balett window coefficients are found via expression:

$$w[n] = \begin{cases} \dfrac{2n}{N-1}; & 0 \le n \le \dfrac{N-1}{2} \\ 2 - \dfrac{2n}{N-1}; & \dfrac{N+1}{2} \le n \le N-1 \end{cases}$$

w[n]={ 0, 0.142857, 0.285714, 0.428571, 0.571429, 0.714286, 0.857143,
1,
0.857143, 0.714286, 0.571429, 0.428571, 0.285714, 0.142857, 0 }

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \dfrac{\sin(\omega_{c2}(n-M))}{\pi(n-M)} - \dfrac{\sin(\omega_{c1}(n-M))}{\pi(n-M)}; & n \ne M \\ \dfrac{\omega_{c2} - \omega_{c1}}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N}{2} = 7$$

Normalized cut-off frequencies ωc1 and ωc2 can be calculated using expressions:

$$\omega_{c1} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 3000}{20000} = 0.3\pi$$

$$\omega_{c2} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 5500}{20000} = 0.55\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc1 and ωc2 with expression for the impulse response coefficients of the ideal band-pass filter:

h_d[n]={ -0.034696, -0.011737, 0.108678, 0.093549, -0.127326, -0.200547, 0.056873,
0.25,
0.056873, -0.200547, -0.127326, 0.093549, 0.108678, -0.011737, -0.034696 }

Step 6:

The designed FIR filter coefficients are found via expression:

h[n] = w[n] * hd[n] ; 0 ≤ n ≤ 14

The FIR filter coefficients h[n] rounded to 6 digits are:

h[n]= { 0, -0.001677, 0.031051, 0.040092, -0.072758, -0.143248, 0.048748,
0.25,
0.048748, -0.143248, -0.072758, 0.040092, 0.031051, -0.001677, 0 }

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-13 illustrates the direct realization of designed FIR filter, whereas figure 2-4-14 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

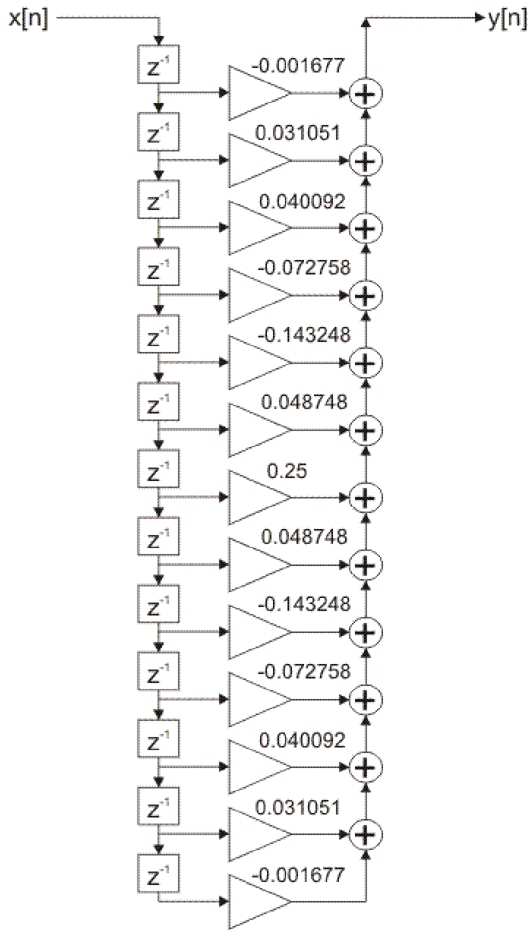


Figure 2-4-13. FIR filter direct realization

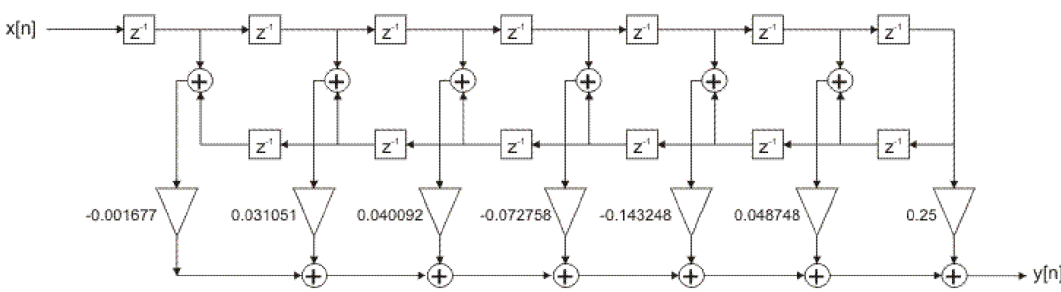


Figure 2-4-14. FIR filter optimized realization structure

2.4.2.4 Example 4

Step 1:

Type of filter – band-stop filter
Filter specifications:

- Filter order – Nf=14;
- Sampling frequency – fs=20KHz; and
- Stopband cut-off frequencies – fc1=3KHz, fc2=5.5KHz.

Step 2:

Method – filter design using Bartlett window

Step 3:

Filter order is predetermined, Nf=14;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=15; and
Coefficients have indices between 0 and 14.

Step 4:

The coefficients of Bartlett window are found via expression:

$$w[n] = \begin{cases} \frac{2n}{N-1}; & 0 \le n \le \frac{N-1}{2} \\ 2 - \frac{2n}{N-1}; & \frac{N+1}{2} \le n \le N-1 \end{cases}$$

w[n]={ 0, 0.142857, 0.285714, 0.428571, 0.571429, 0.714286, 0.857143,
1,
0.857143, 0.714286, 0.571429, 0.428571, 0.285714, 0.142857, 0 }

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \frac{\sin(\omega_{c1}(n-M))}{\pi(n-M)} - \frac{\sin(\omega_{c2}(n-M))}{\pi(n-M)}; & n \ne M \\ 1 - \frac{\omega_{c2} - \omega_{c1}}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N}{2} = 7$$

Normalized cut-off frequencies ωc1 and ωc2 can be calculated using expressions:

$$\omega_{c1} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 3000}{20000} = 0.3\pi$$

$$\omega_{c2} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 5500}{20000} = 0.55\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M, ωc1 and ωc2 with expression for the impulse response coefficients of the ideal band-stop filter:

$h_d[n]$={ 0.034696, 0.011737, -0.108678, -0.093549, 0.127326, 0.200547, -0.056873,
0.75,
-0.056873, 0.200547, 0.127326, -0.093549, -0.108678, 0.011737, 0.034696 }

Note that, excepting the middle element, all the coefficients are the same as in the previous example (band-pass filter with the same cut-off frequencies), but have the opposite sign.

Step 6:

The designed FIR filter coefficients are found via expression:

h[n] = w[n] * hd[n] ; 0 ≤ n ≤ 14

The FIR filter coefficients h[n] rounded to 6 digits are:

h[n]= { 0, 0.001677, -0.031051, -0.040092, 0.072758, 0.143248, -0.048748,
0.750000,
-0.048748, 0.143248, 0.072758, -0.040092, -0.031051, 0.001677, 0}

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-15 illustrates the direct realization of designed FIR filter, whereas figure 2-4-16 illustrates optimized realization of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

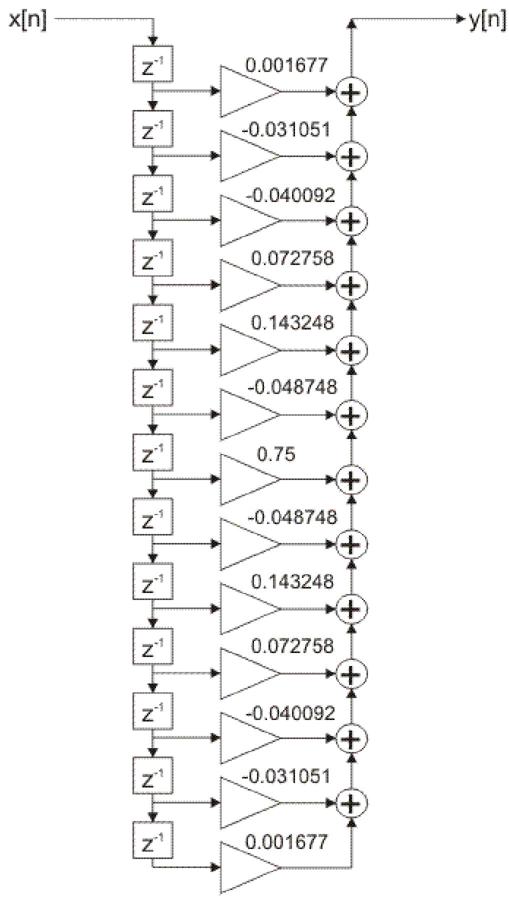


Figure 2-4-15. FIR filter direct realization

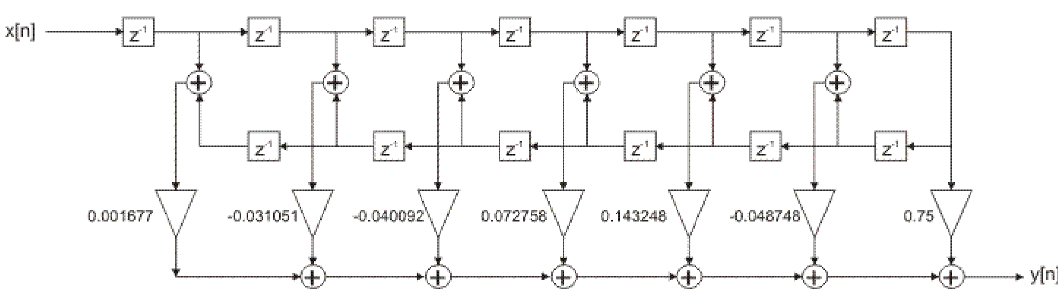


Figure 2-4-16. FIR filter optimized realization structure

It is determined on purpose that FIR filters, explained in examples 3 and 4, have the same order. The similarity between the coefficients of band-pass and band-stop FIR filters is obvious. All coefficients of the band-stop FIR filter have the same absolute values as the corresponding coefficients of the band-pass FIR filter. The only difference is that they are of the opposite sign. The middle element of the band-stop filter is defined as:

$$bbs = 1 - bbp$$

where:

- bbs is the middle coefficient of the band-stop filter; and
- bbp is the middle coefficient of the band-pass filter.

Because of such similarity, it is easy to convert a band-pass FIR filter into a band-stop FIR filter having the same cut-off frequencies, sampling frequency and filter order.

Besides, low-pass and high-pass FIR filters are interrelated in the same way, which can be seen in examples describing Hann window.

2.4.3 Filter design using Hann window

2.4.3.1 Example 1

Step 1:

Type of filter – low-pass filter
Filter specifications:

- Filter order – Nf=10;
- Sampling frequency – fs=20KHz; and
- Passband cut-off frequency – fc=2.5KHz.

Step 2:

Method – filter design using Hann window

Step 3:

Filter order is predetermined, Nf=10;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=11; and
Coefficients have indices between 0 and 10.

Step 4:

The Hann window function coefficients are found via expression:

$$w[n] = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi n}{N-1}\right)\right];\ \ 0 \le n \le N-1$$

w[n]={ 0, 0.095492, 0.345492, 0.654508, 0.904508,
1,
0.904508, 0.654508, 0.345492, 0.095492, 0 }

Step 5:

The ideal low-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \dfrac{\sin[\omega_c(n-M)]}{\pi(n-M)}; & n \ne M \\ \dfrac{\omega_c}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N_f}{2} = 5$$

Normalized cut-off frequency ωc can be calculated using expression:

$$\omega_c = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 2500}{20000} = 0.25\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc with expression for the impulse reaponse coefficients of the ideal low-pass filter:

$h_d[n]$={ -0.045016, 0, 0.075026, 0.159155, 0.225079,
0.25,
0.225079, 0.159155, 0.075026, 0, -0.045016}

Step 6:

The designed FIR filter coefficients are found via expression:

$$h[n] = w[n] * hd[n]\ ;\ 0 \le n \le 10$$

The FIR filter coefficients h[n] rounded to 6 digits are:

$$h[n]=\{\ 0,\ 0,\ 0.025921,\ 0.104168,\ 0.203586,\ 0.25,\ 0.203586,\ 0.104168,\ 0.025921,\ 0,\ 0\ \}$$

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-17 illustrates the direct realization of designed FIR filter, whereas figure 2-4-18 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

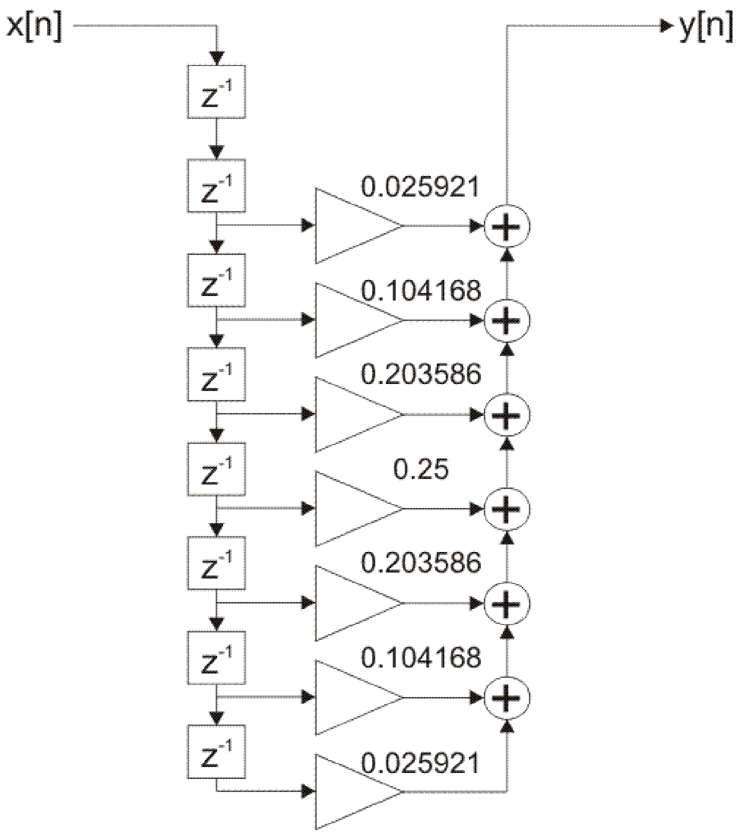


Figure 2-4-17. FIR filter direct realization

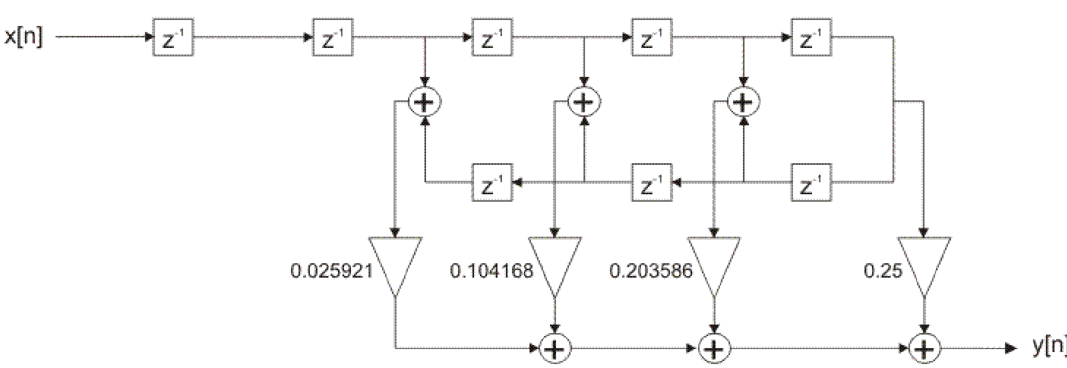


Figure 2-4-18. FIR filter optimized realization structure

2.4.3.2 Example 2

Step 1:

Filter type – high-pass filter Filter specifications:

- Filter order – Nf=10;

- Sampling frequency – fs=20KHz; and
- Passband cut-off frequency – fc=2.5KHz.

Step 2:

Method –filter design using Hann window

Step 3:

Filter order is predetermined, Nf=10;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=11; and
Coefficients have indices between 0 and 10.

Step 4:

The Hann window function coefficients are found via expression:

$$w[n] = \frac{1}{2}\left[1 - \cos(\frac{2\pi n}{N-1})\right]; \ 0 \le n \le N-1$$

$$w[n]=\{\ 0,\ 0.095492,\ 0.345492,\ 0.654508,\ 0.904508,\ 1,\ 0.904508,\ 0.654508,\ 0.345492,\ 0.095492,\ 0\ \}$$

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} 1 - \frac{\omega_c}{\pi}; & n \neq M \\ -\frac{\sin(\omega_c(n-M))}{\pi(n-M)}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N}{2} = 5$$

Normalized cut-off frequency ωc can be calculated using expression:

$$\omega_c = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 2500}{20000} = 0.25\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc with expression for the impulse response coefficients of the ideal high-pass filter:

$$h_d[n]=\{\ 0.045016,\ 0,\ -0.075026,\ -0.159155,\ -0.225079,\ 0.75,\ -0.225079,\ -0.159155,\ -0.075026,\ 0,\ 0.045016\}$$

Step 6:

The designed FIR filter coefficients are found via expression:

$$h[n] = w[n] * hd[n] ;\ 0 \le n \le 10$$

The FIR filter coefficients h[n] rounded to 6 digits are:

$$h[n]=\{\ 0,\ 0,\ -0.025921,\ -0.104168,\ -0.203586,\ 0.75,\ -0.203586,\ -0.104168,\ -0.025921,\ 0,\ 0\}$$

Step 7:

The filter order is predetermined.

There is no need to additionally change it.

Filter realization:

Figure 2-4-19 illustrates the direct realization of designed FIR filter, whereas figure 2-4-20 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

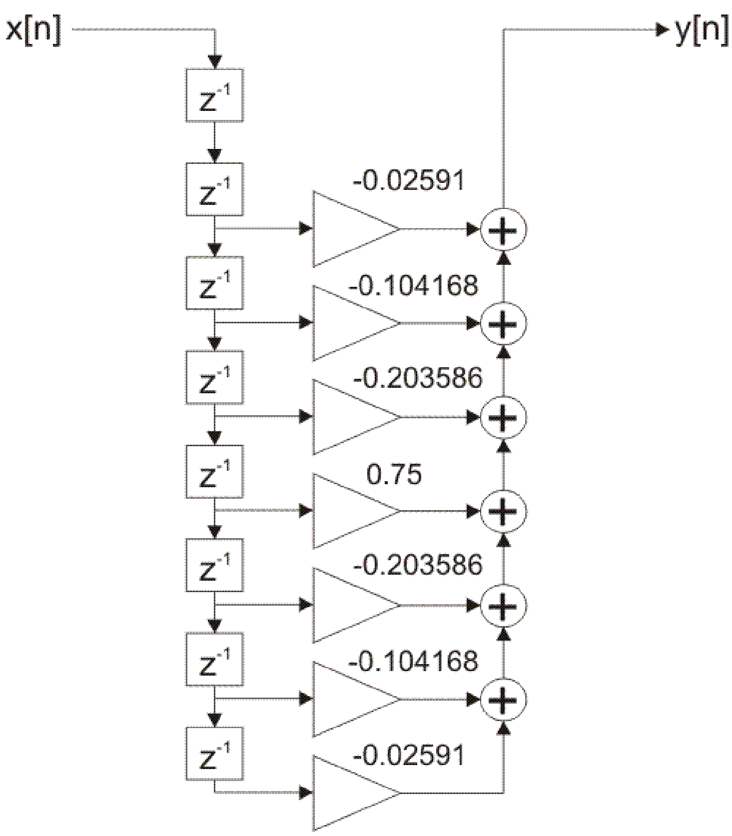


Figure 2-4-19. FIR filter direct realization

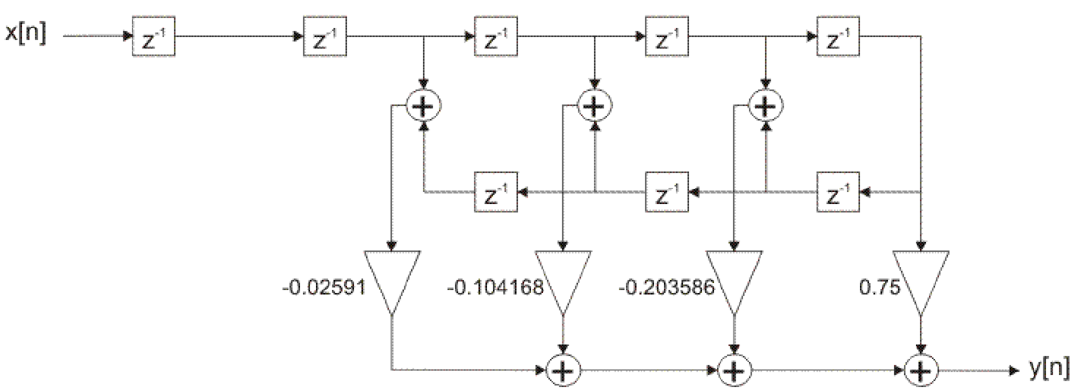


Figure 2-4-20. FIR filter optimized realization structure

2.4.3.3 Example 3

Step 1:

Type of filter – band-pass filter
Filter specifications:

- Filter order – Nf=14;
- Sampling frequency – fs=20KHz; and
- Passband cut-off frequency – fc1=3KHz, fc2=5.5KHz.

Step 2:

Method – filter design using Hann window

Step 3:

Filter order is predtermined, Nf=14;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=15; and
Coefficients have indices between 0 and 14.

Step 4:

The Hann window function coefficients are found via expression:

$$w[n] = \frac{1}{2}\left[1-\cos(\frac{2\pi n}{N-1})\right];\ \ 0 \le n \le N-1$$

w[n]={ 0, 0.049516, 0.188255, 0.38874, 0.61126, 0.811745, 0.950484,
1,
0.950484, 0.811745, 0.61126, 0.38874, 0.188255, 0.049516, 0 }

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \dfrac{\sin(\omega_{c2}(n-M))}{\pi(n-M)} - \dfrac{\sin(\omega_{c1}(n-M))}{\pi(n-M)}; & n \ne M \\ \dfrac{\omega_{c2}-\omega_{c1}}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N}{2} = 7$$

Normalized cut-off frequencies ωc1 and ωc2 can be calculated using expressions:

$$\omega_{c1} = \frac{2\pi f_c}{f_s} = \frac{2\cdot\pi\cdot 3000}{20000} = 0.3\pi$$

$$\omega_{c2} = \frac{2\pi f_c}{f_s} = \frac{2\cdot\pi\cdot 5500}{20000} = 0.55\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M, ωc1 and ωc2 with expression for the impulse response coefficients of the ideal band-pass filter:

h_d[n]={ -0.034696, -0.011737, 0.108678, 0.093549, -0.127326, -0.200547, 0.056873,
0.25,
0.056873, -0.200547, -0.127326, 0.093549, 0.108678, -0.011737, -0.034696 }

Step 6:

The designed FIR filter coefficients are found via expression:

h[n] = w[n] * hd[n] ; 0 ≤ n ≤ 14

The FIR filter coefficients h[n] rounded to 6 digits are:

h[n]= { 0, -0.000581, 0.020459, 0.036366, -0.07783, -0.162793, 0.054057,
0.25,
0.054057, -0.162793, -0.07783, 0.036366, 0.020459, -0.000581, 0 }

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-21 illustrates the direct realization of designed FIR filter, whereas figure 2-4-22 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

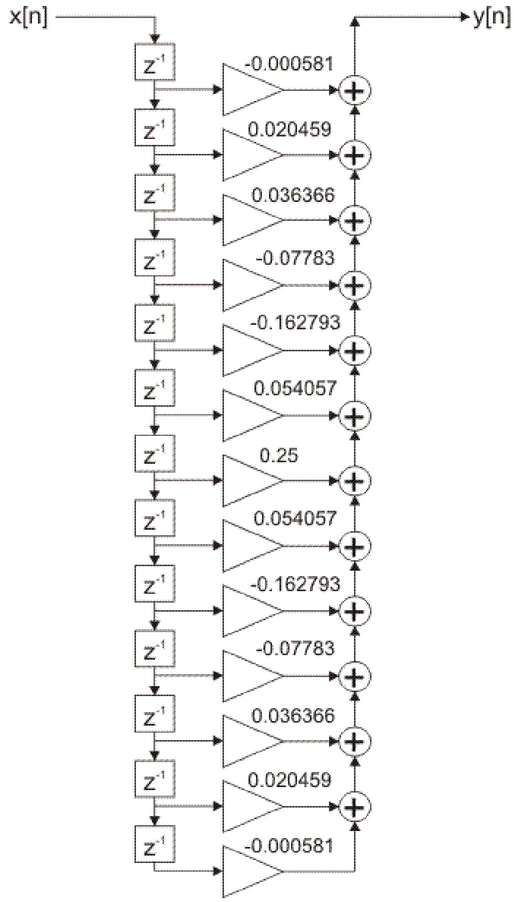


Figure 2-4-21. FIR filter direct realization

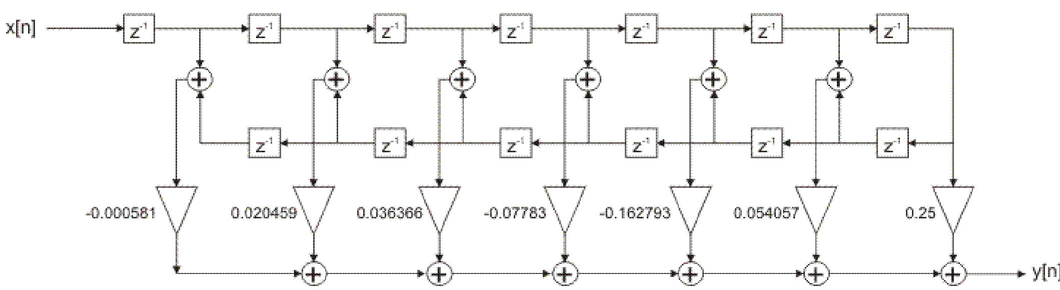


Figure 2-4-22. FIR filter optimized realization structure

2.4.3.3 Example 3

Step 1:

Type of filter – band-stop filter
Filter specifications:

- Filter order – Nf=14;
- Sampling frequency – fs=20KHz; and
- Passband cut-off frequency – fc1=3KHz, fc2=5.5KHz.

Step 2:

Method – filter design using Hann window

Step 3:

Filter order is predetermined, Nf=14;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=15;
Coefficients have indices between 0 and 14.

Step 4:

The Hann window function coefficients are found via expression:

$$w[n] = \frac{1}{2}\left[1 - \cos(\frac{2\pi n}{N-1})\right];\ \ 0 \le n \le N-1$$

w[n]={ 0, 0.049516, 0.188255, 0.38874, 0.61126, 0.811745, 0.950484,
1,
0.950484, 0.811745, 0.61126, 0.38874, 0.188255, 0.049516, 0 }

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \frac{\sin(\omega_{c1}(n-M))}{\pi(n-M)} - \frac{\sin(\omega_{c2}(n-M))}{\pi(n-M)}; & n \ne M \\ 1 - \frac{\omega_{c2} - \omega_{c1}}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N}{2} = 7$$

Normalized cut-off frequencies ωc1 and ωc2 can be calculated using expressions:

$$\omega_{c1} = \frac{2\pi f_c}{f_s} = \frac{2\cdot\pi\cdot 3000}{20000} = 0.3\pi$$

$$\omega_{c2} = \frac{2\pi f_c}{f_s} = \frac{2\cdot\pi\cdot 5500}{20000} = 0.55\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M, ωc1 and ωc2 with expression for the impulse response coefficients of the ideal band-stop filter:

h_d[n]={ 0.034696, 0.011737, -0.108678, -0.093549, 0.127326, 0.200547, -0.056873,
0.75,
-0.056873, 0.200547, 0.127326, -0.093549, -0.108678, 0.011737, 0.034696 }

Note that, excepting the middle element, all coefficients are the same as in the previous example (band-pass filter with the same cut-off frequencies), but have the opposite sign.

Step 6:

The designed FIR filter coefficients are found via expression:

h[n]=w[n] * hd[n] ; 0 ≤ n ≤ 14

The FIR filter coefficients h[n] rounded to 6 digits are:

h[n]= { 0, 0.000581, -0.020459, -0.036366, 0.07783, 0.162793, -0.054057,
0.75,
-0.054057, 0.162793, 0.07783, -0.036366, -0.020459, 0.000581, 0 }

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-23 illustrates the direct realization of designed FIR filter, whereas figure 2-4-24 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

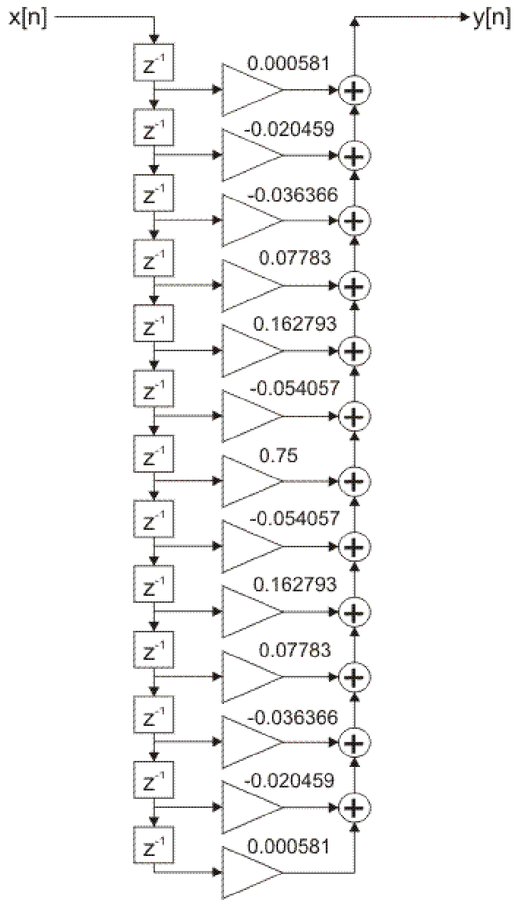


Figure 2-4-23. FIR filter direct realization

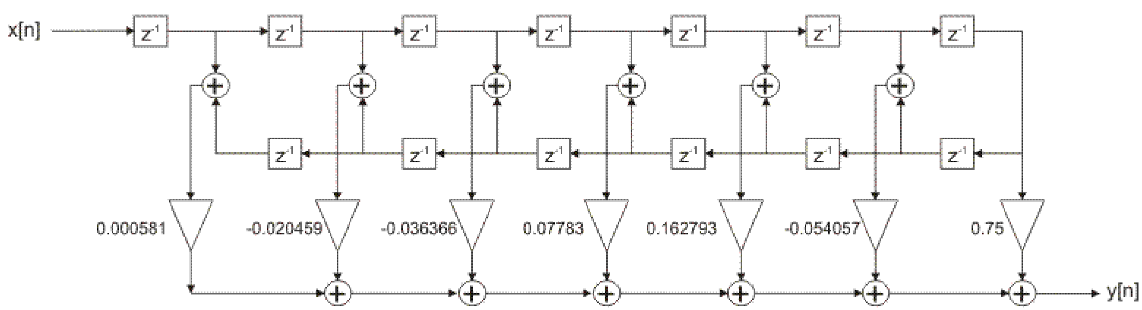


Figure 2-4-24. FIR filter optimized realization structure

It is specified on purpose that FIR filters, explained in examples 1 and 2, have the same order. The similarity between low-pass and high-pass FIR filter coefficients is obvious. All coefficients of the low-pass FIR filter have the same absolute values as the corresponding coefficients of the high-pass FIR filter. The only difference is that they are of the opposite sign. The middle element is defined as:

$$blp = 1 - bhp$$

where:

- blp is the middle coefficient of a low-pass filter; and
- bhp is the middle coefficient of a high-pass filter.

Because of such similarity, it is easy to convert a low-pass FIR filter into a high-pass FIR filter having the same cut-off frequencies, sampiling frequency and filter order.

2.4.4 Filter design using Bartlett-Hanning window

2.4.4.1 Example 1

Step 1:

Type of filter – low-pass filter
Filter specifications:

- Filter order – Nf=9;
- Sampling frequency – fs=22050Hz; and
- Passband cut-off frequency – fc=4KHz.

Step 2:

Method – filter design using Bartlett-Hanning window

Step 3:

Filter order is predetermined, Nf=9;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=10; and
Coefficients have indices between 0 and 9.

Step 4:

The Bartlett-Hanning window function coefficients are found via expression:

$$w[n] = 0.62 - 0.48\left|\frac{n}{N-1} - 0.5\right| + 0.38\cos(2\pi(\frac{n}{N-1} - 0.5)); 0 \le n \le N-1$$

$$w[n]=\{ 0, 0.142236, 0.42068, 0.73, 0.950417, 0.950417, 0.73, 0.42068, 0.142236, 0 \}$$

Step 5:

The ideal low-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \frac{\sin[\omega_c(n-M)]}{\pi(n-M)}; & n \ne M \\ \frac{\omega_c}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N_f}{2} = 4.5$$

Normalized cut-off frequency ωc can be calculated using expression:

$$\omega_c = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 4000}{22050} = 0.3628\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc with expression for the impulse response coefficients of the ideal low-pass filter:

$$h_d[n]=\{ -0.064681, -0.068189, 0.036662, 0.210162, 0.343489, 0.343489, 0.210162, 0.036662, -0.068189, -0.064681 \}$$

Step 6:

The designed FIR filter coefficients are found via expression:

$$h[n] = w[n] * hd[n] ; 0 \le n \le 9$$

The FIR filter coefficients h[n] rounded to 6 digits are:

$$h[n]=\{ 0, -0.009699, 0.015423, 0.153419, 0.326457, 0.326457, 0.153419, 0.015423, -0.009699, 0 \}$$

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-25 illustrates the direct realization of designed FIR filter, whereas figure 2-4-26 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

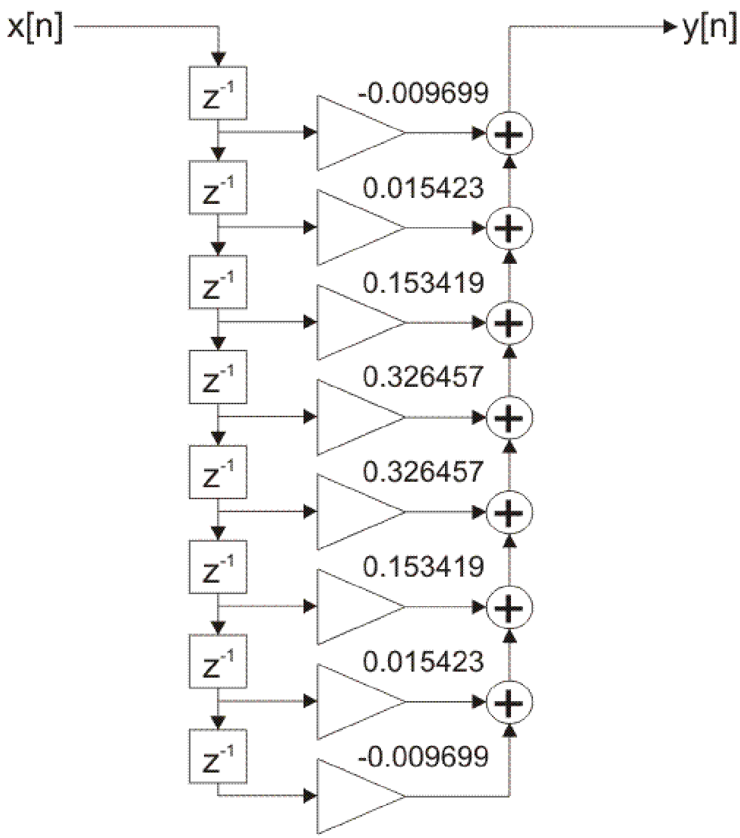


Figure 2-4-25. FIR filter direct realization

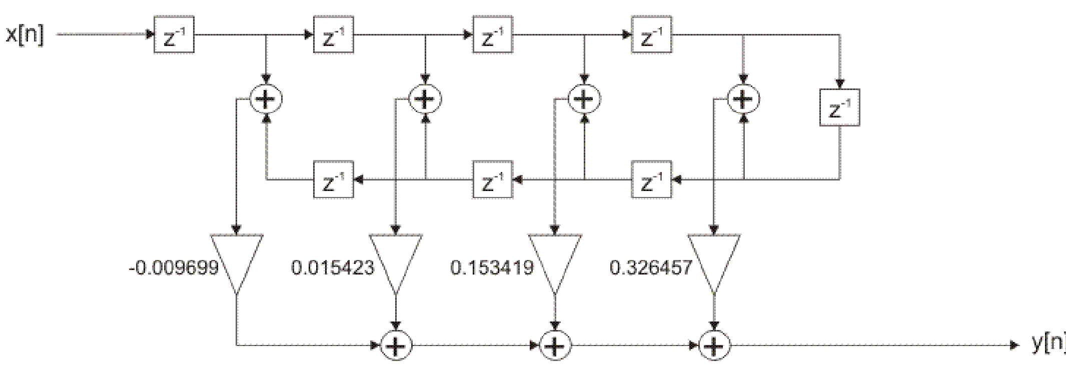


Figure 2-4-26. FIR filter optimized realization structure

2.4.4.2 Example 2

Step 1:

Type of filter – high-pass filter
Filter specifications:

- Filter order – Nf=10;
- Sampling frequency – fs=22050Hz; and
- Passband cut-off frequency – fc=4KHz.

Step 2:

Method – filter design using Bartlett-Hanning window

Step 3:

Filter order is predetermined, Nf=10;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=11; and
Coefficients have indices between 0 and 10.

Step 4:

The Bartlett-Hanning window function coefficients are found via expression:

$$w[n] = 0.62 - 0.48\left|\frac{n}{N-1} - 0.5\right| + 0.38\cos(2\pi(\frac{n}{N-1} - 0.5)); 0 \le n \le N-1$$

w[n]={ 0, 0.120574, 0.358574, 0.641426, 0.879426,
1,
0.879426, 0.641426, 0.358574, 0.120574, 0 }

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} 1 - \frac{\omega_c}{\pi}; & n \ne M \\ -\frac{\sin(\omega_c(n-M))}{\pi(n-M)}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N}{2} = 5$$

Normalized cut-off frequency ωc may be calculated using expression:

$$\omega_c = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 4000}{22050} = 0.3628\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc with expression for the impulse response coefficients of the ideal high-pass filter:

h_d[n]={ 0, 0.075683, 0.062366, -0.093549, -0.302731,
0.6,
-0.302731, -0.093549, 0.062366, 0.075683, 0 }

Step 6:

The designed FIR filter coefficients are found via expression:

h[n] = w[n] * hd[n] ; 0 ≤ n ≤ 10

The FIR filter coefficients h[n] rounded to 6 digits are:

h[n]={ 0, 0.009125, 0.022363, -0.060005, -0.266229,
0.6,
-0.266229, -0.060005, 0.022363, 0.009125, 0 }

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-27 illustrates the direct realization of designed FIR filter, whereas figure 2-4-28 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

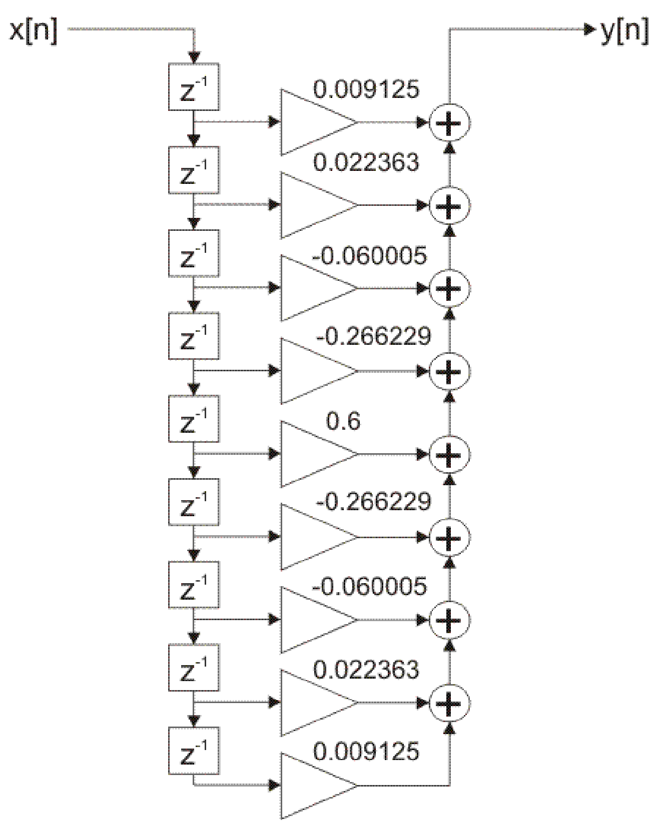


Figure 2-4-27. FIR filter direct realization

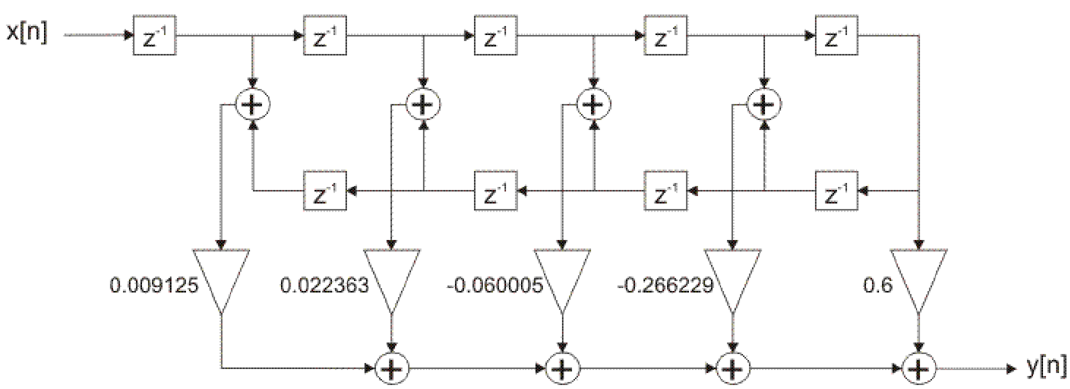


Figure 2-4-28. FIR filter optimized realization structure

2.4.4.3 Example 3

Step 1:

Type of filter – band-pass filter
Filter specifications:

- Filter order – Nf=12;
- Sampling frequency – fs=22050Hz; and
- Passband cut-off frequency – fc1=2KHz, fc2=5KHz.

Step 2:

Method – filter design using Bartlett-Hanning window

Step 3:

Filter order is predetermined, Nf=12;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=13; and
Coefficients have indices between 0 and 12.

Step 4:

The Bartlett-Hanning window function coefficients are found via expression:

$$w[n] = 0.62 - 0.48\left|\frac{n}{N-1} - 0.5\right| + 0.38\cos(2\pi(\frac{n}{N-1} - 0.5)); 0 \le n \le N-1$$

w[n]={ 0, 0.09091, 0.27, 0.5, 0.73, 0.90909,
1,
0.90909, 0.73, 0.5, 0.27, 0.09091, 0 }

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \frac{\sin(\omega_{c2}(n-M))}{\pi(n-M)} - \frac{\sin(\omega_{c1}(n-M))}{\pi(n-M)}; & n \ne M \\ \frac{\omega_{c2} - \omega_{c1}}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N}{2} = 6$$

Normalized cut-off frequencies ωc1 and ωc2 can be calculated using expressions:

$$\omega_{c1} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 2000}{22050} = 0.1814\pi$$

$$\omega_{c2} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 5000}{22050} = 0.4535\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M, ωc1 and ωc2 with expression for the impulse response coefficients of the ideal band-pass filter:

h_d[n]={ 0.055311, 0.0291, -0.104296, -0.201163, -0.098774, 0.143177,
0.272109,
0.143177, -0.098774, -0.201163, -0.104296, 0.0291, 0.055311 }

Step 6:

The designed FIR filter coefficients are found via expression:

h[n] = w[n] * hd[n] ; 0 ≤ n ≤ 12

The FIR filter coefficients h[n] rounded to 6 digits are:

h[n]= { 0, 0.002646, -0.02816, -0.100582, -0.072105, 0.130161,
0.272109,
0.130161, -0.072105, -0.100582, -0.02816, 0.002646, 0 }

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-29 illustrates the direct realization of designed FIR filter, whereas figure 2-4-30 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

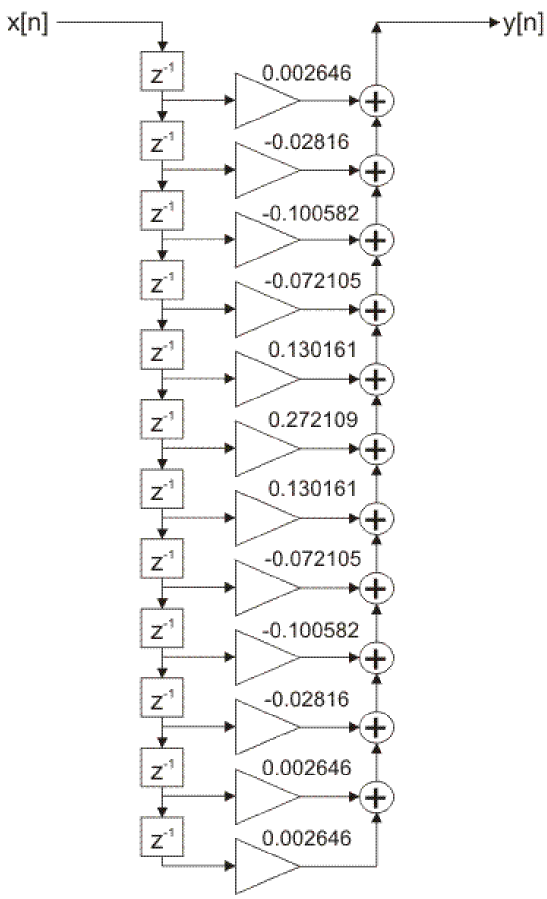


Figure 2-4-29. FIR filter direct realization

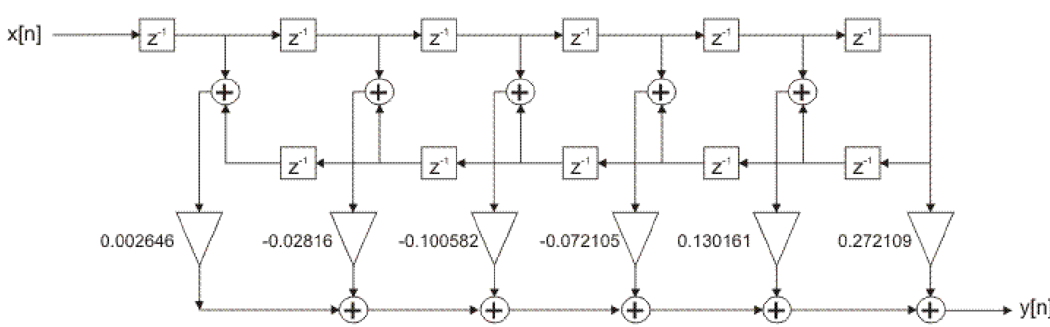


Figure 2-4-30. FIR filter optimized realization structure

2.4.4.4 Example 4

Step 1:

Type of filter – band-stop filter
Filter specifications:

- Filter order – Nf=12;
- Sampling frequency – fs=22050Hz; and
- Passband cut-off frequencies – fc1=2KHz, fc2=6KHz.

Step 2:

Method – filter design using Bartlett-Hanning window

Step 3:

Filter order is predetermined, Nf=12;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=13; and
Coefficients have indices between 0 and 12.

Step 4:

The Bartlett-Hanning window function coefficients are found via expression:

$$w[n] = 0.62 - 0.48\left|\frac{n}{N-1} - 0.5\right| + 0.38\cos(2\pi(\frac{n}{N-1} - 0.5)); 0 \le n \le N-1$$

w[n]={ 0, 0.09091, 0.27, 0.5, 0.73, 0.90909,
1,
0.90909, 0.73, 0.5, 0.27, 0.09091, 0 }

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \frac{\sin(\omega_{c1}(n-M))}{\pi(n-M)} - \frac{\sin(\omega_{c2}(n-M))}{\pi(n-M)}; & n \ne M \\ 1 - \frac{\omega_{c2} - \omega_{c1}}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N}{2} = 6$$

Normalized cut-off frequencies ωc1 and ωc2 can be calculated using expressions:

$$\omega_{c1} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 2000}{22050} = 0.1814\pi$$

$$\omega_{c2} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 6000}{22050} = 0.5442\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M, ωc1 and ωc2 with expression for the impulse reaponse coefficients of the ideal band-stop filter:

h_d[n]={ 0.024723, -0.030582, 0.018433, 0.202103, 0.188252, -0.143499,
0.637188,
-0.143499, 0.188252, 0.202103, 0.018433, -0.030582, 0.024723 }

Step 6:

The designed FIR filter coefficients are found via expression:

h[n] = w[n] * hd[n] ; 0 ≤ n ≤ 12

The FIR filter coefficients h[n] rounded to 6 digits are:

h[n]= { 0, -0.00278, 0.004977, 0.101052, 0.137424, -0.130454,
0.637188,
-0.130454, 0.137424, 0.101052, 0.004977, -0.00278, 0 }

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-31 illustrates the direct realization of designed FIR filter, whereas figure 2-4-32 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

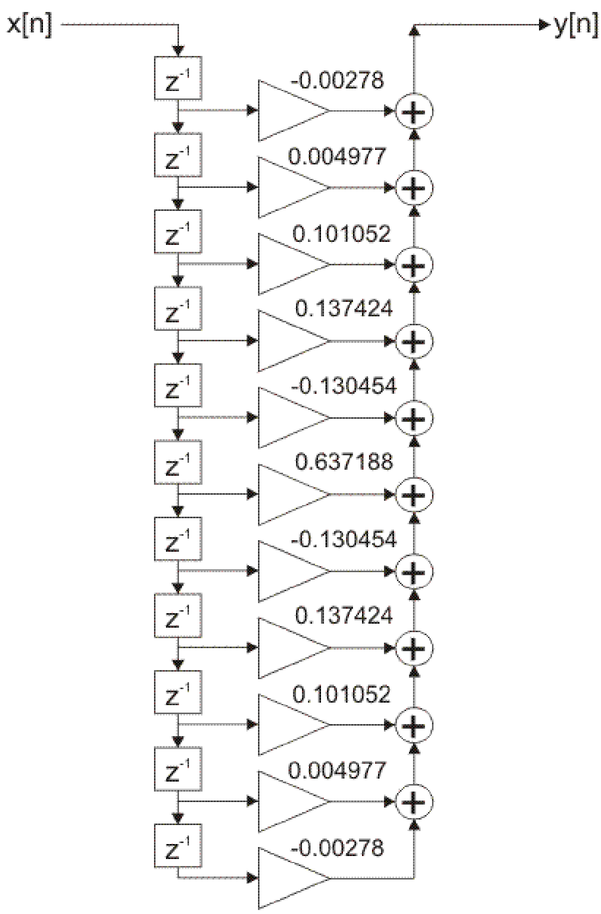


Figure 2-4-31. FIR filter direct realization

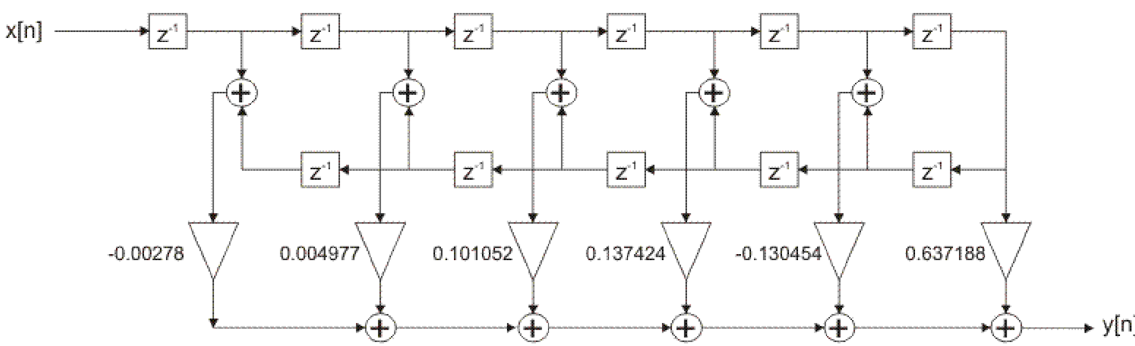


Figure 2-4-32. FIR filter optimized realization structure

2.4.5 Filter design using Hamming window

2.4.5.1 Example 1

Step 1:

Type of filter – low-pass filter
Filte specifications:

- Sampling frequency – fs=22050Hz;
- Passband cut-off frequency – fc1=3KHz;
- Stopband cut-off frequency – fc2=6KHz; and
- Minimum stopband attenuation – 40dB.

Step 2:

Method – filter design using Hamming window

Step 3:

For the first iteration, the filter order can be determined from the table 2-4-1 below.

WINDOW FUNCTION	NORMALIZED LENGTH OF THE MAIN LOBE FOR N=20	TRANSITION REGION FOR N=20	MINIMUM STOPBAND ATTENUATION OF WINDOW FUNCTION	MINIMUM STOPBAND ATTENUATION OF DESIGNED FILTER
Rectangular	0.1π	0.041π	13 dB	21 dB
Triangular (Bartlett)	0.2π	0.11π	26 dB	26 dB
Hann	0.21π	0.12π	31 dB	44 dB
Bartlett-Hanning	0.21π	0.13π	36 dB	39 dB
Hamming	**0.23π**	**0.14π**	**41 dB**	**53 dB**
Bohman	0.31π	0.2π	46 dB	51 dB
Blackman	0.32π	0.2π	58 dB	75 dB
Blackman-Harris	0.43π	0.32π	91 dB	109 dB

Table 2-4-1. Comparison of window functions

Using the specifications for the transition region of the required filter, it is possible to compute cut-off frequencies:

$$\omega_{c1} = \frac{2 \cdot \pi \cdot 3000}{22050} = 0.2721\pi$$

$$\omega_{c2} = \frac{2 \cdot \pi \cdot 6000}{22050} = 0.5442\pi$$

The required transition region of the filter is:

$$\Delta\omega = \omega_{c2} - \omega_{c1} = 0.2721\pi$$

The transition region of the filter to be designed is approximately twice that of the filter given in the table above. For the first iteration, the filter order can be half of that.

- Filter order is Nf=10;
- A total number of filter coefficients is larger by 1, i.e. N=Nf+1=11; and
- Coefficients have indices between 0 and 10.

Step 4:

The Hamming window function coefficients are found via expression:

$$w[n] = 0.54 - 0.46(1 - \cos(\frac{2\pi n}{N-1})); 0 \le n \le N-1$$

$$w[n]=\{ 0.08, 0.167852, 0.397852, 0.682148, 0.912148, 1, 0.912148, 0.682148, 0.397852, 0.167852, 0.08 \}$$

Step 5:

The ideal low-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \dfrac{\sin[\omega_c(n-M)]}{\pi(n-M)}; & n \ne M \\ \dfrac{\omega_c}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N_f}{2} = 5$$

Normalized cut-off frequency ωc can be calculated using expression:

$$\omega_c = \omega_{c1} = 0.2721\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc with expression for the impulse response coefficients of the ideal low-pass filter:

$$h_d[n]=\{ -0.057649, -0.021826, 0.057883, 0.157622, 0.240157, 0.272109, 0.240157, 0.157622, 0.057883, -0.021826, -0.057649 \}$$

Step 6:

The designed FIR filter coefficients are found via expression:

h[n] = w[n] * hd[n] ; 0 ≤ n ≤ 10

The FIR filter coefficients h[n] rounded to 6 digits are:

$$h[n]=\{ -0.004612, -0.003663, 0.023029, 0.107521, 0.219059, 0.272109, 0.219059, 0.107521, 0.023029, -0.003663, -0.004612 \}$$

Step 7:

Analyse in the frequency domain is performed using the Filter Designer Tool program.

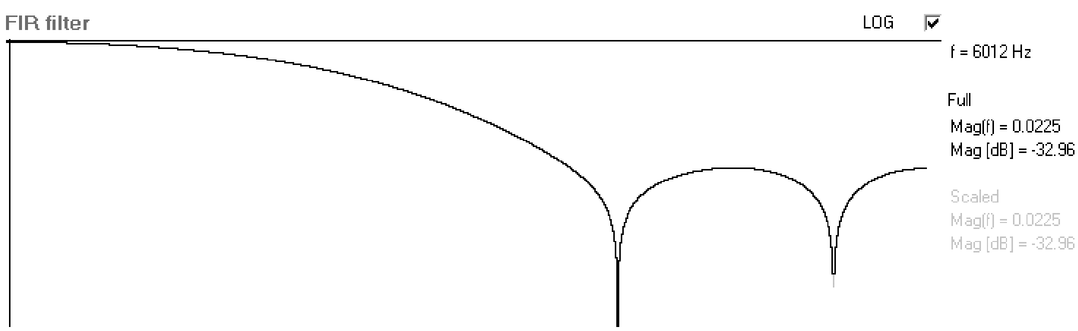


Figure 2-4-33. Frequency characteristic of the resulting filter

Figure 2-4-33 illustrates the frequency characteristic of the resulting filter. It is obtained in the **Filter Designer Tool** program. As seen, the resulting filter doesn't satisfy the required specifications. The attenuation at the frequency of 6KHz amounts to 32.96dB only, which is not sufficient. It is necessary to increase the filter order.

Another way is to compute the attenuation at the frequency of 6KHz. Starting from the impulse response, the first thing that should be done is the Z-transform. It is explained, along with Fourier transformation, in chapter 2-2-2.

$$H(z) = \sum_{n=0}^{10} h[n] \cdot z^{-n} = -0.004612 - 0.003663 \cdot z^{-1} + 0.023029 \cdot z^{-2} + 0.107521 \cdot z^{-3} + 0.219059 \cdot z^{-4} + 0.272109 \cdot z^{-5} + 0.219059 \cdot z^{-6} + 0.107521 \cdot z^{-7} + 0.023029 \cdot z^{-8} - 0.003663 \cdot z^{-9} - 0.004612 \cdot z^{-10}$$

It is easy to obtain the Fourier transformation via the Z-transform:

$$H(e^{j\omega}) = H(z = e^{j\omega})$$

$$\omega = \frac{2 \cdot \pi \cdot 6000}{22050} = 0.5442$$

$$H(e^{j0.5442\pi}) = -0.004612 - 0.003663 \cdot e^{-j0.5442\pi} + 0.023029 \cdot e^{-j1.0884\pi} + 0.107521 \cdot e^{-j1.6326\pi} + 0.219059 \cdot e^{-j2.1768\pi} + 0.272109 \cdot e^{-j2.721\pi} + 0.219059 \cdot e^{-j3.2652\pi} + 0.107521 \cdot e^{-j3.8094\pi} + 0.023029 \cdot e^{-j4.3536\pi} - 0.003663 \cdot e^{-j4.8978\pi} - 0.004612 \cdot e^{-j5.442\pi}$$

$$\left|H(e^{j0.5442\pi})\right| = [-0.004612 - 0.003663\cos(-0.5442\pi) + 0.023029\cos(-1.0884\pi) + 0.107521\cos(-1.6326\pi) + 0.219059\cos(-2.1768\pi) + 0.272109\cos(-2.721\pi) + 0.219059\cos(-3.2652\pi) + 0.107521\cos(-3.8094\pi) + 0.023029\cos(-4.3536\pi) - 0.003663\cos(-4.8978\pi) - 0.004612\cos(-5.442\pi)]^2 + [-0.003663\sin(-0.5442\pi) + 0.023029\sin(-1.0884\pi) + 0.107521\sin(-1.6326\pi) + 0.219059\sin(-2.1768\pi) + 0.272109\sin(-2.721\pi) + 0.219059\sin(-3.2652\pi) + 0.107521\sin(-3.8094\pi) + 0.023029\sin(-4.3536\pi) - 0.003663\sin(-4.8978\pi) - 0.004612\sin(-5.442\pi)]^2$$

$$= 0.02297921$$

$$20\log(0.02297921) = -32.7733\text{dB}$$

According to the analyse performed using Filter Designer Tool, it is confirmed that the filter order has to be incremented.

The filter order is incremented by two. The whole process of designing filter is repeated from the step 3.

Step 3:

Filter order is Nf=12;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=13; and
Coefficients have indices between 0 and 12.

Step 4:

The Hamming window function coefficients are found via expression:

$$w[n] = 0.54 - 0.46(1 - \cos(\frac{2\pi n}{N-1})); 0 \le n \le N-1$$

$$w[n] = \{ 0.08, 0.141628, 0.31, 0.54, 0.77, 0.938372, 1, 0.938372, 0.77, 0.54, 0.31, 0.141628, 0.08 \}$$

Step 5:

The ideal low-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \frac{\sin[\omega_c(n-M)]}{\pi(n-M)}; & n \ne M \\ \frac{\omega_c}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N_f}{2} = 6$$

Normalized cut-off frequency ωc can be calculated using expression:

$$\omega_c = \omega_{c1} = 0.2721\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc with expression for the impulse response coefficients of the ideal low-pass filter:

$$h_d[n]=\{ -0.048511, -0.057649, -0.021826, 0.057883, 0.157622, 0.240157, 0.272109, 0.240157, 0.157622, 0.057883, -0.021826, -0.057649, -0.048511 \}$$

Step 6:

The designed FIR filter coefficients are found via expression:

$$h[n] = w[n] * hd[n] ;\ 0 \le n \le 12$$

The FIR filter coefficients h[n] rounded to 6 digits are:

$$h[n]=\{ -0.003881, -0.008165, -0.006766, 0.031257, 0.121369, 0.225357, 0.272109, 0.225357, 0.121369, 0.031257, -0.006766, -0.008165, -0.003881\}$$

Step 7:

Analyse in the frequency domain is performed using the Filter Designer Tool program.

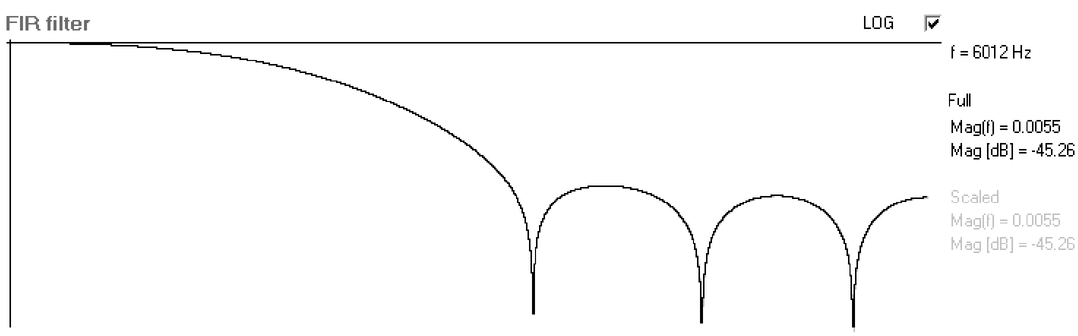


Figure 2-4-34. Frequency characteristic of the resulting filter

Figure 2-4-34 illustrates the frequency characteristic of the resulting filter. As seen, the resulting filter doesn't satisfy the given specifications. The attenuation at the frequency of 6KHz amounts to 45.26dB only, which is not sufficient. It is necessary to change the filter order.

Filter realization:

Figure 2-4-35 illustrates the direct realization of designed FIR filter, whereas figure 2-4-36 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

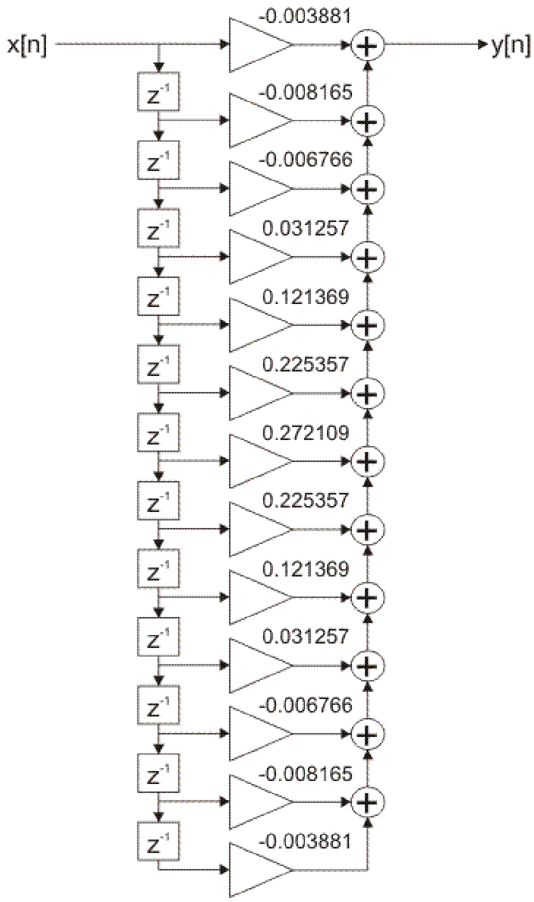


Figure 2-4-35. FIR filter direct realization

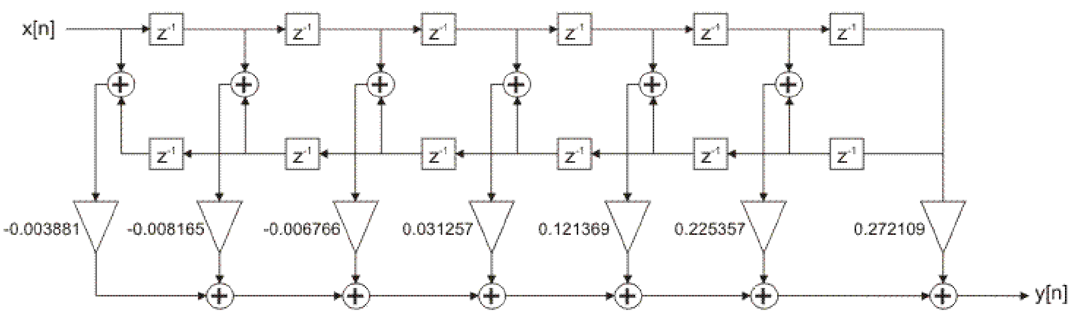


Figure 2-4-36. FIR filter optimized realization structure

2.4.5.2 Example 2

Step 1:

Type of filter – high-pass filter
Filte specifications:

- Filter order – Nf=10;
- Sampling frequency – fs=22050Hz; and
- Passband cut-off frequency – fc=4KHz.

Step 2:

Method – filter design using Hamming window

Step 3:

Filter order is predetermined, Nf=10;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=11; and
Coefficients have indices between 0 and 10.

Step 4:

The Hamming window function coefficients are found via expression:

$$w[n] = 0.54 - 0.46(1 - \cos(\frac{2\pi n}{N-1})); 0 \le n \le N-1$$

$$w[n] = \{ 0.08, 0.167852, 0.397852, 0.682148, 0.912148, \\ 1, \\ 0.912148, 0.682148, 0.397852, 0.167852, 0.08 \}$$

The Hamming window function is one of rare standard windows where w[0] > 0 is in effect.

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} 1 - \frac{\omega_c}{\pi}; & n \neq M \\ -\frac{\sin(\omega_c(n-M))}{\pi(n-M)}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N_f}{2} = 5$$

Normalized cut-off frequency ωc can be calculated using expression:

$$\omega_c = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 4000}{22050} = 0.3628\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc with expression for the impulse response coefficients of the ideal high-pass filter:

$$h_d[n] = \{ 0.035109, 0.078646, 0.029101, -0.120820, -0.289201, \\ 0.637188, \\ -0.289201, -0.120820, 0.029101, 0.078646, 0.035109 \}$$

Step 6:

The designed FIR filter coefficients are found via expression:

$$h[n] = w[n] * hd[n] ; 0 \le n \le 10$$

The FIR filter coefficients h[n] rounded to 6 digits are:

$$h[n] = \{ 0.002809, 0.013201, 0.011578, -0.082417, -0.263794, \\ 0.637188, \\ -0.263794, -0.082417, 0.011578, 0.013201, 0.002809 \}$$

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-37 illustrates the direct realization of designed FIR filter, whereas figure 2-4-38 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

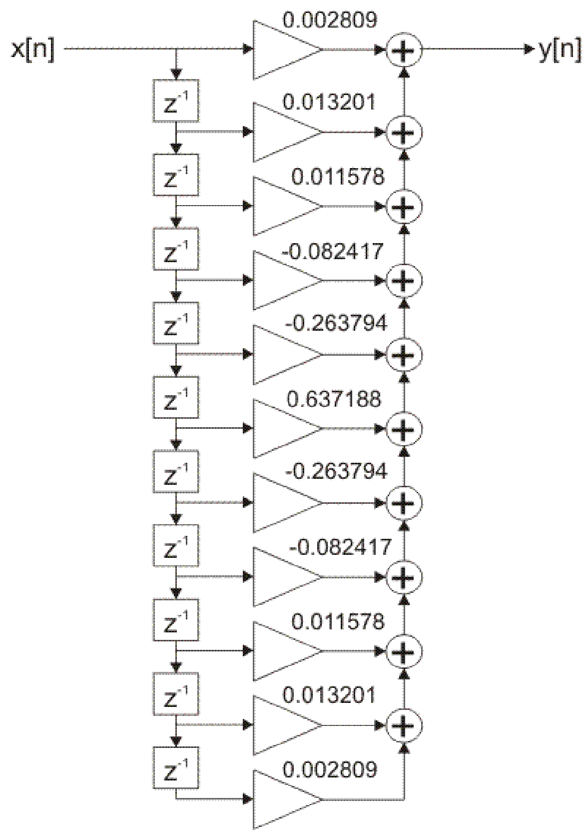


Figure 2-4-37. FIR filter direct realization

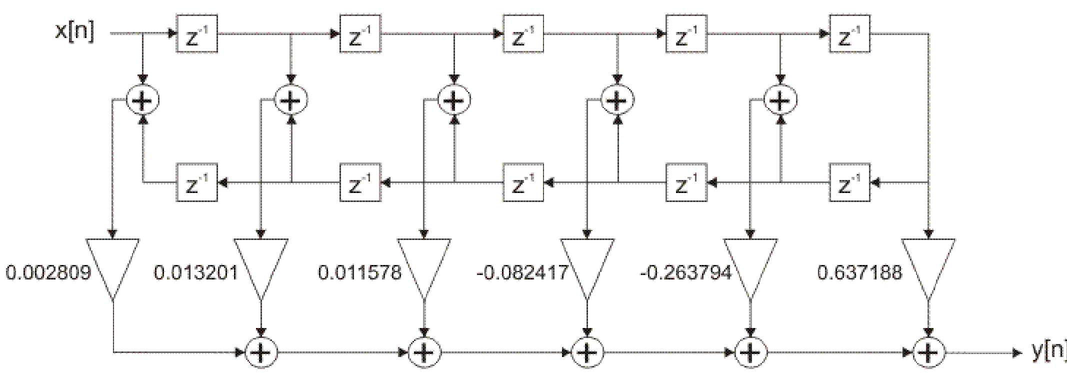


Figure 2-4-38. FIR filter optimized realization structure

2.4.5.3 Example 3

Step 1:

Type of filter – band-pass filter
Filte specifications:

- Filter order – Nf=12;
- Sampling frequency – fs=16KHz;
- Passband cut-off frequency – fc1=2KHz, fc2=5KHz.

Step 2:

Method – filter design using Hamming window

Step 3:

Filter order is predetermined, Nf=12;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=13; and
Coefficients have indices between 0 and 12.

Step 4:

The Hamming window function coefficients are found via expression:

$$w[n] = 0.54 - 0.46(1 - \cos(\frac{2\pi n}{N-1})); 0 \le n \le N-1$$

$$w[n] = \{ 0.08, 0.141628, 0.31, 0.54, 0.77, 0.938372, \\ 1, \\ 0.938372, 0.77, 0.54, 0.31, 0.141628, 0.08 \}$$

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \frac{\sin(\omega_{c2}(n-M))}{\pi(n-M)} - \frac{\sin(\omega_{c1}(n-M))}{\pi(n-M)}; & n \neq M \\ \frac{\omega_{c2} - \omega_{c1}}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N_f}{2} = 6$$

Normalized cut-off frequencies ωc1 and ωc2 can be calculated using expressions:

$$\omega_{c1} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 2000}{16000} = 0.25\pi$$

$$\omega_{c2} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 5000}{16000} = 0.625\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M, ωc1 and ωc2 with expression for the impulse response coefficients of the ideal band-pass filter:

$$h_d[n] = \{ 0.015538, 0.020653, 0.079577, -0.115630, -0.271694, 0.069001, \\ 0.375, \\ 0.069001, -0.271694, -0.115630, 0.079577, 0.020653, 0.015538 \}$$

Step 6:

The designed FIR filter coefficients are found via expression:

h[n] = w[n] * hd[n] ; 0 ≤ n ≤ 12

The FIR filter coefficients h[n] rounded to 6 digits are:

$$h[n] = \{ 0.001243, 0.002925, 0.024669, -0.06244, -0.209205, 0.064749, \\ 0.375, \\ 0.064749, -0.209205, -0.062440, 0.024669, 0.002925, 0.001243 \}$$

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-39 illustrates the direct realization of designed FIR filter, whereas figure 2-4-40 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle elements.

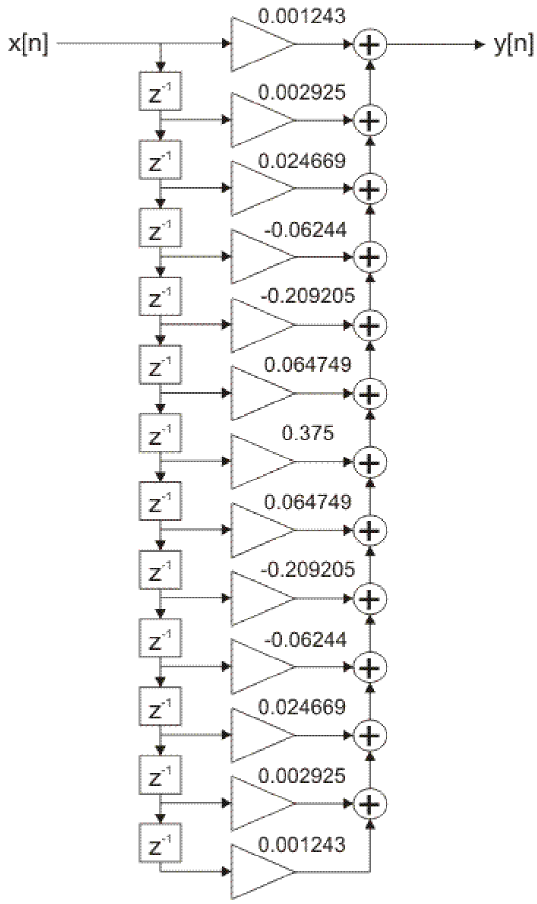


Figure 2-4-39. FIR filter direct realization

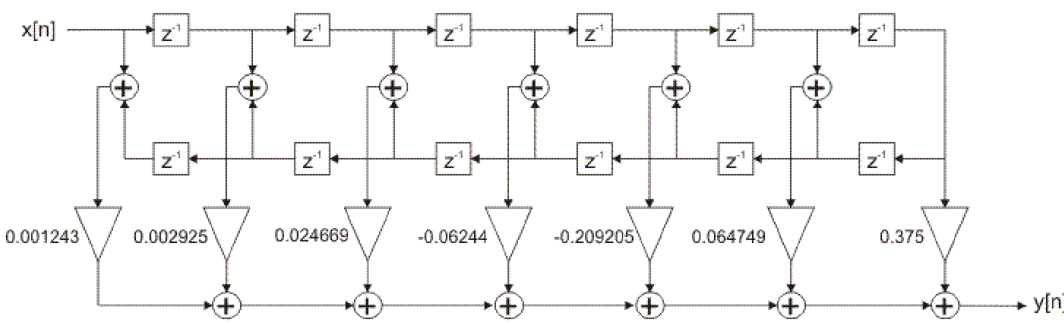


Figure 2-4-40. FIR filter optimized realization structure

2.4.5.4 Example 4

Step 1:

Type of filter – band-stop filter
Filte specifications:

- Filter order – Nf=12;
- Sampling frequency – fs=16000Hz; and
- Passband cut-off frequency – fc1=2KHz, fc2=6KHz.

Step 2:

Method – filter design using Hamming window

Step 3:

Filter order is predetermined, Nf=12;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=13; and
Coefficients have indices between 0 and 12.

Step 4:

The Bartlett-Hanning window function coefficients are found via expression:

$$w[n] = 0.54 - 0.46(1 - \cos(\frac{2\pi n}{N-1})); 0 \le n \le N-1$$

$$w[n]=\{ 0.08, 0.141628, 0.31, 0.54, 0.77, 0.938372, 1, 0.938372, 0.77, 0.54, 0.31, 0.141628, 0.08 \}$$

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \frac{\sin(\omega_{c1}(n-M))}{\pi(n-M)} - \frac{\sin(\omega_{c2}(n-M))}{\pi(n-M)}; & n \ne M \\ 1 - \frac{\omega_{c2} - \omega_{c1}}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N_f}{2} = 6$$

Normalized cut-off frequencies ωc1 and ωc2 can be calculated using expressions:

$$\omega_{c1} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 2000}{16000} = 0.25\pi$$

$$\omega_{c2} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 6000}{16000} = 0.75\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M, ωc1 and ωc2 with expression for the impulse response coefficients of the ideal band-stop filter:

$$h_d[n]=\{ -0.106103, 0, 0, 0, 0.31831, 0, 0.5, 0, 0.31831, 0, 0, 0, -0.106103 \}$$

Step 6:

The designed FIR filter coefficients are found via expression:

h[n] = w[n] * hd[n] ; 0 ≤ n ≤ 12

The FIR filter coefficients h[n] rounded to 6 digits are:

$$h[n]= \{ -0.008488, 0, 0, 0, 0.245099, 0, 0.5, 0, 0.245099, 0, 0, 0, -0.008488 \}$$

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-41 illustrates the direct realization of designed FIR filter, whereas figure 2-4-42 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

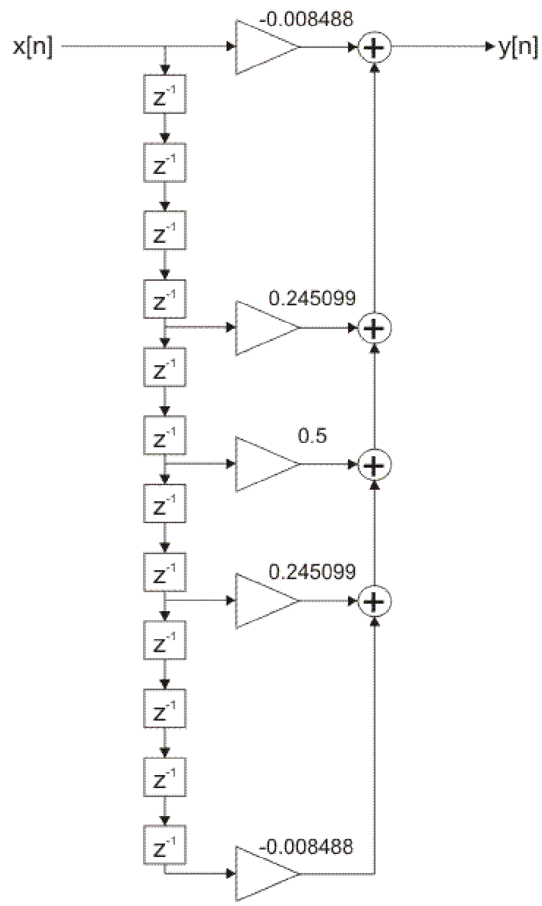


Figure 2-4-41. FIR filter direct realization

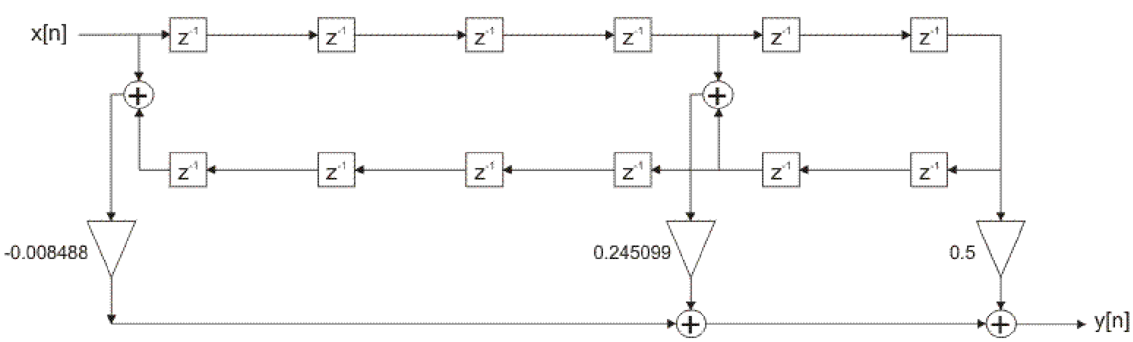


Figure 2-4-42. FIR filter optimized realization structure

The first example (low-pass filter designed using Hamming window) explains the algorithm used to compute the needed filter order when it is unknown. The filter order can also be found using Kaiser window, after which the number of iterations, i.e. correction steps is reduced.

The forth example explains the way of designing a band-stop filter. As can be seen, the impulse response of the resulting filter contains large number of zero values, which results in reducing the number of multiplication operations in design process. These zeros appear in impulse response because of the stopband width which amounts to 0.5π = π/2.

If it is possible to specify the sampling frequency from a certain frequency range, you should tend to specify the value representing a multiple of the passband width. The number of zeros contained in an impulse response is larger in this case, whereas the number of multiplications, otherwise the most demanding operation in filtering process, is less.

In the given example, only 5 multiplication operations are performed in direct realization of a twelfth-order FIR filter, i.e. 3 multiplication operations in optimized realization structure.

2.4.6 Filter design using Bohman window

2.4.6.1 Example 1

Step 1:

Type of filter – low-pass filter

Filte specifications:

- Filter order – Nf=10;
- Sampling frequency – fs=20KHz; and
- Passband cut-off frequency – fc=5KHz.

Step 2:

Method – filter design using Bohman window

Step 3:

Filter order is Nf=10;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=11; and
Coefficients have indices between 0 and 10.

Step 4:

The Bohman window function coefficients are found via expression:

$$w[n] = \left(1 - \frac{\left|n - \frac{N-1}{2}\right|}{\frac{N-1}{2}}\right)\cos\left(\pi \frac{\left|n - \frac{N-1}{2}\right|}{\frac{N-1}{2}}\right) + \frac{1}{\pi}\sin\left(\pi \frac{\left|n - \frac{N-1}{2}\right|}{\frac{N-1}{2}}\right); 0 \le n \le N-1$$

$$\begin{aligned} w[n]=\{ & 0, 0.025294, 0.179124, 0.488141, 0.834311, \\ & 1, \\ & 0.834311, 0.488141, 0.179124, 0.025294, 0 \} \end{aligned}$$

Step 5:

The ideal low-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \frac{\sin[\omega_c(n-M)]}{\pi(n-M)}; & n \neq M \\ \frac{\omega_c}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N_f}{2} = 5$$

Normalized cut-off frequency ωc may be computed using expression:

$$\omega_c = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 5000}{20000} = 0.5\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc with expression for the impulse response coefficients of the ideal low-pass filter:

$$\begin{aligned} h_d[n]=\{ & 0.063662, 0, -0.106103, 0, 0.31831, \\ & 0.5, \\ & 0.31831, 0, -0.106103, 0, 0.063662 \} \end{aligned}$$

Step 6:

The designed FIR filter coefficients are found via expression:

$$h[n] = w[n] * hd[n] ; 0 \le n \le 10$$

The FIR filter coefficients h[n] rounded to 6 digits are:

$$h[n]=\{\ 0,\ 0,\ -0.019006,\ 0,\ 0.26557,\ 0.5,\ 0.26557,\ 0,\ -0.019006,\ 0,\ 0\ \}$$

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-43 illustrates the direct realization of designed FIR filter, whereas figure 2-4-44 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

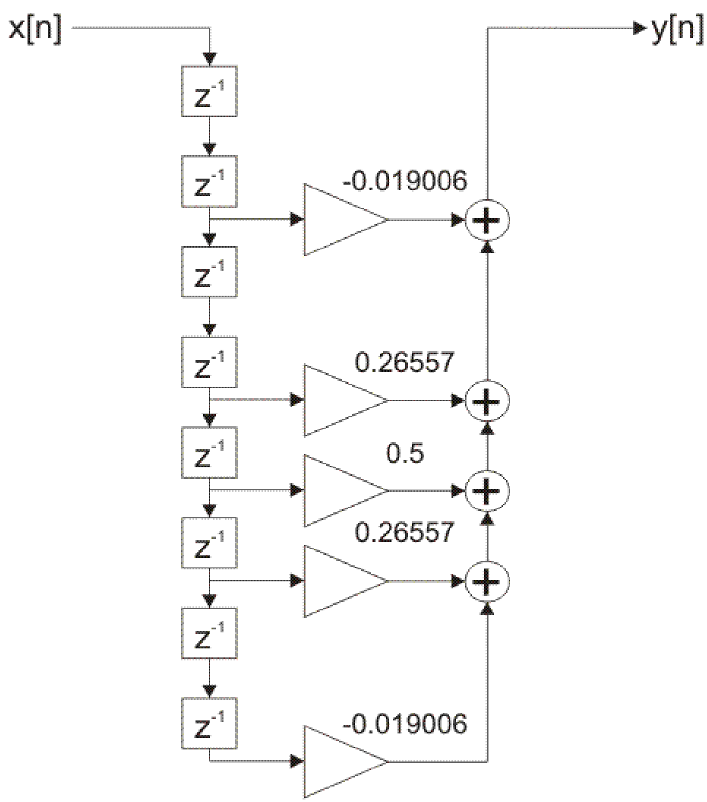


Figure 2-4-43. FIR filter direct realization

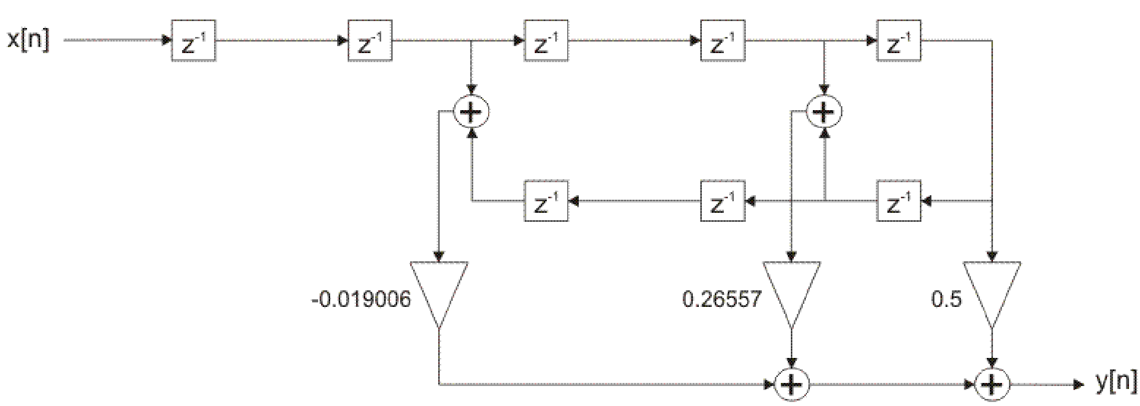


Figure 2-4-44. FIR filter optimized realization structure

2.4.6.2 Example 2

Step 1:

Type of filter – high-pass filter

Filte specifications:

- Sampling frequency – fs=22050Hz;
- Passband cut-off frequency – fc1=1.5KHz;
- Stopband cut-off frequency – fc2=4KHz; and
- Minimum stopband attenuation – 35dB.

Step 2:

Method – filter design using Bohman window

Step 3:

The needed filter order is determined via iteration.
It is necessary to specify the initial value of filter order that is to be changed as many times as needed. This value is specified according to the data contained in the table 2-4-2 below:

WINDOW FUNCTION	NORMALIZED LENGTH OF THE MAIN LOBE FOR N=20	TRANSITION REGION FOR N=20	MINIMUM STOPBAND ATTENUATION OF WINDOW FUNCTION	MINIMUM STOPBAND ATTENUATION OF DESIGNED FILTER
Rectangular	0.1π	0.041π	13 dB	21 dB
Triangular (Bartlett)	0.2π	0.11π	26 dB	26 dB
Hann	0.21π	0.12π	31 dB	44 dB
Bartlett-Hanning	0.21π	0.13π	36 dB	39 dB
Hamming	0.23π	0.14π	41 dB	53 dB
Bohman	**0.31π**	**0.2π**	**46 dB**	**51 dB**
Blackman	0.32π	0.2π	58 dB	75 dB
Blackman-Harris	0.43π	0.32π	91 dB	109 dB

Table 2-4-2. Comparison of window functions

According to the specifications for the transition region of required filter, it is possible to compute cut-off frequencies:

$$\omega_{c1} = \frac{2 \cdot \pi \cdot 1500}{22050} = 0.1361\pi$$

$$\omega_{c2} = \frac{2 \cdot \pi \cdot 4000}{22050} = 0.3628\pi$$

The required transition region is:

$$\Delta\omega = \omega_{c2} - \omega_{c1} = 0.2267\pi$$

The transition region of the filter to be designed is somewhat wider than that of the filter given in table 2-4-2. For the first iteration, during filter design process, the filter order can be lower.

Unlike the low-pass FIR filter, the high-pass FIR filter must be of even order. The same applies to band-pass and band-stop filters. It means that filter order can be changed in odd steps. The smallest change is ±2. In this case, the filter order, comparing to that from the table (20), can be decreased by 2 for the purpose of defining initial value.

Filter order is Nf=18;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=19; and
Coefficients have indices between 0 and 18.

Step 4:

The coefficients of Bohman window are found via expression:

$$w[n] = \left(1 - \frac{\left|n - \frac{N-1}{2}\right|}{\frac{N-1}{2}}\right)\cos\left(\pi\frac{\left|n - \frac{N-1}{2}\right|}{\frac{N-1}{2}}\right) + \frac{1}{\pi}\sin\left(\pi\frac{\left|n - \frac{N-1}{2}\right|}{\frac{N-1}{2}}\right); 0 \le n \le N-1$$

$$w[n]=\{ 0, 0.004458, 0.034374, 0.108998, 0.236297, 0.409945, 0.608998, 0.800418, 0.944151, 1.000000, 0.944151, 0.800418, 0.608998, 0.409945, 0.236297, 0.108998, 0.034374, 0.004458, 0 \}$$

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} 1-\dfrac{\omega_c}{\pi}; & n \neq M \\ -\dfrac{\sin(\omega_c(n-M))}{\pi(n-M)}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N_f}{2} = 9$$

Normalized cut-off frequency ωc is equal to passband cut-off frequency:

$$\omega_c=\omega_{c2}=0.3628\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc with expression for the impulse response coefficients of the ideal high-pass filter:

$$h_d[n]=\{ 0.026182, -0.011998, -0.045120, -0.027985, 0.035109, 0.078646, 0.029101, -0.120820, -0.289201, 0.637188, -0.289201, -0.120820, 0.029101, 0.078646, 0.035109, -0.027985, -0.045120, -0.011998, 0.026182 \}$$

Step 6:

The designed FIR filter coefficients are found via expression:

h[n] = w[n] * hd[n] ; 0 ≤ n ≤ 18

The FIR filter coefficients h[n] rounded to 6 digits are:

$$h[n]=\{ 0, -0.000053, -0.001551, -0.003050, 0.008296, 0.032241, 0.017722, -0.096706, -0.273050, 0.637188, -0.273050, -0.096706, 0.017722, 0.032241, 0.008296, -0.003050, -0.001551, -0.000053, 0 \}$$

Step 7:

Analyse in the frequency domain is performed using the **Filter Designer Tool** program.

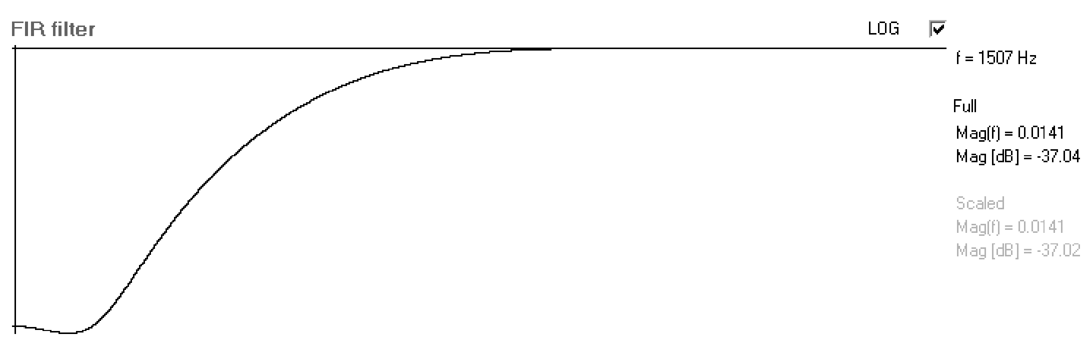


Figure 2-4-45. Frequency characteristics of the resulting filter

Figure 2-4-45 illustrates the frequency characteristic of the resulting filter. It is obtained in the **Filter Designer Tool** program. As seen, the resulting filter satisfies the required specifications. The attenuation at the frequency of 1500Hz amounts to 37.04dB only, which is more than enough. However, the final objective when designing a filter is to find a minimum filter order that satisfies the filter specifications.

Since the filter order must be changed by an even number, the specified value is -2. The filter order is decreased by 2. The whole process of designing filters is repeated from the step 3 on.

Step 3:

Filter order is Nf=16;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=17; and
Coefficients have indices between 0 and 16.

Step 4:

The coefficients of Bohman window are found via expression:

$$w[n] = \left(1 - \frac{\left|n - \frac{N-1}{2}\right|}{\frac{N-1}{2}}\right)\cos\left(\pi\frac{\left|n - \frac{N-1}{2}\right|}{\frac{N-1}{2}}\right) + \frac{1}{\pi}\sin\left(\pi\frac{\left|n - \frac{N-1}{2}\right|}{\frac{N-1}{2}}\right); 0 \le n \le N-1$$

$$\begin{aligned} w[n]=\{ & 0,\ 0.006327,\ 0.048302,\ 0.150574,\ 0.31831, \\ & 0.533257,\ 0.755409,\ 0.930207, \\ & 1, \\ & 0.930207,\ 0.755409,\ 0.533257, \\ & 0.318310,\ 0.150574,\ 0.048302,\ 0.006327,\ 0 \} \end{aligned}$$

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} 1 - \frac{\omega_c}{\pi}; & n \neq M \\ -\frac{\sin(\omega_c(n-M))}{\pi(n-M)}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N_f}{2} = 8$$

Normalized cut-off frequency ωc is equal to the passband cut-off frequency:

$$\omega_c = \omega_{c2} = 0.3628\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc with expression for the impulse response coefficients of the ideal high-pass filter:

$$\begin{aligned} h_d[n]=\{ & -0.011998,\ -0.045120,\ -0.027985,\ 0.035109,\ 0.078646, \\ & 0.029101,\ -0.120820,\ -0.289201, \\ & 0.637188, \\ & -0.289201,\ -0.120820,\ 0.029101, \\ & 0.078646,\ 0.035109,\ -0.027985,\ -0.045120,\ -0.011998 \} \end{aligned}$$

Step 6:

The designed FIR filter coefficients are found via expression:

$$h[n] = w[n] * hd[n] ;\ 0 \le n \le 16$$

The FIR filter coefficients h[n] rounded to 6 digits are:

h[n]={ 0, -0.000285, -0.001352, 0.005286, 0.025034,
0.015518, -0.091268, -0.269017,
0.637188,
-0.269017, -0.091268, 0.015518,
0.025034, 0.005286, -0.001352, -0.000285, 0 }

Step 7:

Analyse in the frequency domain is performed using the Filter Designer Tool program.

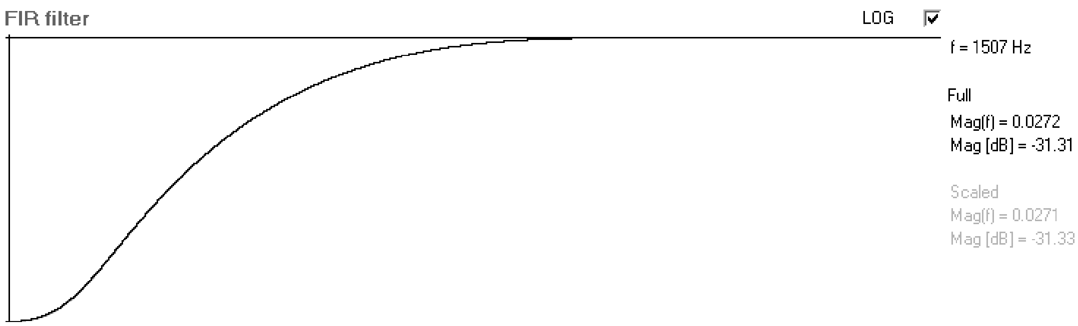


Figure 2-4-46. Frequency characteristic of the resulting filter

Figure 2-4-46 illustrates the frequency characteristic of the resulting filter. The figure is obtained in the **Filter Designer Tool** program. As seen, the resulting filter satisfies the required specifications. The objective is to find the minimum filter order. Since the attenuation is close to the required attenuation, the correct order is probably 16. However, it is necessary to check it.

Since the filter order must be changed by an even number, the specified value is -2. The filter order is decreased by 2, therefore. The whole process of designing filter is repeated from the step 3 on.

Step 3:

Filter order is predetermined, Nf=14;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=15; and
Coefficients have indices between 0 and 14.

Step 4:

The coefficients of Bohman window are found via expression:

$$w[n] = \left(1 - \frac{\left|n - \frac{N-1}{2}\right|}{\frac{N-1}{2}}\right)\cos\left(\pi\frac{\left|n - \frac{N-1}{2}\right|}{\frac{N-1}{2}}\right) + \frac{1}{\pi}\sin\left(\pi\frac{\left|n - \frac{N-1}{2}\right|}{\frac{N-1}{2}}\right); 0 \le n \le N-1$$

w[n]={ 0, 0.0094, 0.070725, 0.214963, 0.437484, 0.694215, 0.910369,
1,
0.910369, 0.694215, 0.437484, 0.214963, 0.070725, 0.0094, 0 }

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} 1 - \frac{\omega_c}{\pi}; & n \neq M \\ -\frac{\sin(\omega_c(n-M))}{\pi(n-M)}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N_f}{2} = 7$$

Normalized cut-off frequency ωc is equal to passband cut-off freqency:

$$\omega_c=\omega_{c2}=0.3628\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc with expression for the impulse reaponse coefficients of the ideal high-pass filter:

$$h_d[n]=\{ -0.04512, -0.027985, 0.035109, 0.078646, 0.029101, -0.12082, -0.289201, 0.637188, -0.289201, -0.12082, 0.029101, 0.078646, 0.035109, -0.027985, -0.04512 \}$$

Step 6:

The designed FIR filter coefficients are found via expression:

$$h[n] = w[n] * hd[n] ;\ 0 \le n \le 14$$

The FIR filter coefficients h[n] rounded to 6 digits are:

$$h[n]=\{ 0, -0.000263, 0.002483, 0.016906, 0.012731, -0.083875, -0.26328, 0.637188, -0.26328, -0.083875, 0.012731, 0.016906, 0.002483, -0.000263, 0 \}$$

Step 7:

Analyse in the frequency domain is performed using the Filter Designer Tool program.

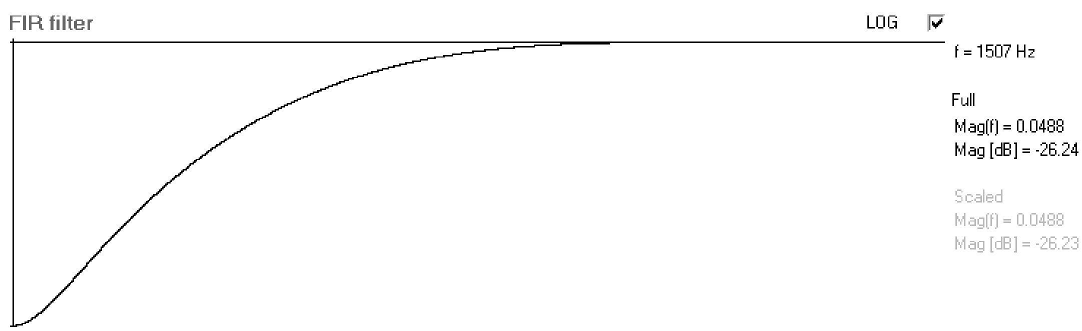


Figure 2-4-47. Frequency characteristic of the resulting filter

Figure 2-4-47 illustrates the frequency characteristic of the resulting filter. The figure is obtained in the Filter Designer Tool program. As seen, the resulting filter doesn't satisfy the required specifications. The attenuation at the frequency of 1500KHz amounts to 26.24dB only, which is not sufficient. The previous value (Nf=16) represents the minimum FIR filter order that satisfies the given specifications.

The filter order is Nf=16, whereas impulse response of the resulting filter is as follows:

$$h[n]=\{ 0, -0.000285, -0.001352, 0.005286, 0.025034, 0.015518, -0.091268, -0.269017, 0.637188, -0.269017, -0.091268, 0.015518, 0.025034, 0.005286, -0.001352, -0.000285, 0 \}$$

Filter realization:

Figure 2-4-48 illustrates the direct realization of designed FIR filter, whereas figure 2-4-49 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

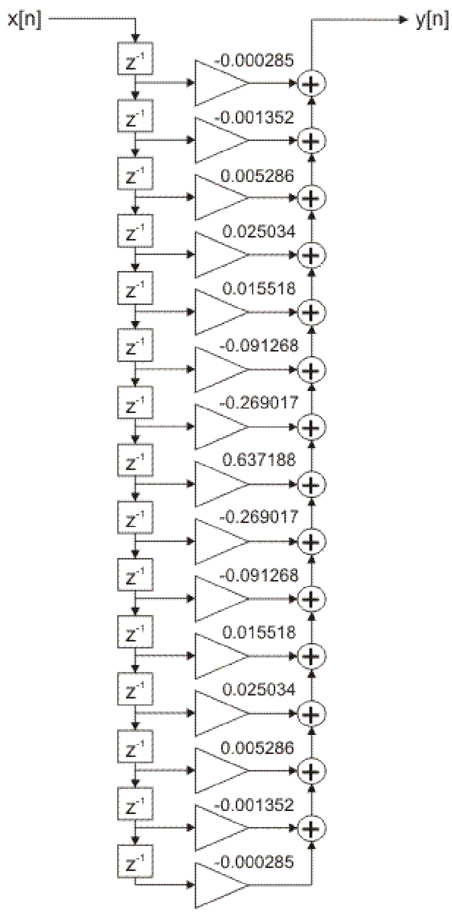


Figure 2-4-48. FIR filter direct realization

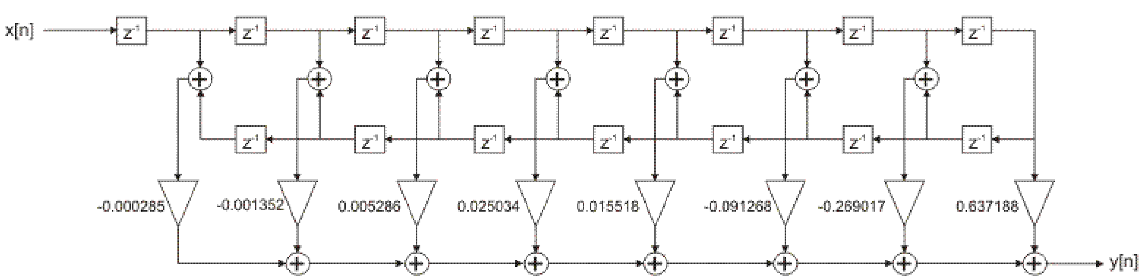


Figure 2-4-49. FIR filter optimized realization structure

2.4.6.3 Example 3

Step 1:

Type of filter – band-pass filter
Filte specifications:

- Filter order – Nf=12;
- Sampling frequency – fs=44100Hz; and
- Passband cut-off frequency – fc1=4KHz, fc2=15025Hz.

Step 2:

Method – filter design using Bohman window

Step 3:

Filter order is predetermined, Nf=12;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=13; and
Coefficients have indices between 0 and 12.

Step 4:

The coefficients of Bohman window are found via expression:

$$w[n] = \left(1 - \frac{\left|n - \frac{N-1}{2}\right|}{\frac{N-1}{2}}\right)\cos\left(\pi\frac{\left|n - \frac{N-1}{2}\right|}{\frac{N-1}{2}}\right) + \frac{1}{\pi}\sin\left(\pi\frac{\left|n - \frac{N-1}{2}\right|}{\frac{N-1}{2}}\right); 0 \le n \le N-1$$

w[n]={ 0, 0.014817, 0.108998, 0.31831, 0.608998, 0.880843,
1,
0.880843, 0.608998, 0.31831, 0.108998, 0.014817, 0 }

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \frac{\sin(\omega_{c2}(n-M))}{\pi(n-M)} - \frac{\sin(\omega_{c1}(n-M))}{\pi(n-M)}; & n \neq M \\ \frac{\omega_{c2} - \omega_{c1}}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N}{2} = 6$$

Normalized cut-off frequencies ωc1 and ωc2 can be calculated using expressions:

$$\omega_{c1} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 4000}{44100} = 0.1814\pi$$

$$\omega_{c2} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 15025}{44100} = 0.6814\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M, ωc1 and ωc2 with expression for the impulse response coefficients of the ideal band-pass filter:

$h_d[n]$={ 0.029101, -0.079297, 0, -0.090389, -0.289201, 0.096258,
0.5,
0.096258, -0.289201, -0.090389, 0, -0.079297, 0.029101 }

Step 6:

The coefficients of designed FIR filter are found via expression:

h[n] = w[n] * hd[n] ; 0 ≤ n ≤ 12

The FIR filter coefficients h[n] rounded to 6 digits are:

h[n]= { 0, -0.001175, 0, -0.028772, -0.176123, 0.084788,
0.5,
0.084788, -0.176123, -0.028772, 0, -0.001175, 0 }

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-50 illustrates the direct realization of designed FIR filter, whereas figure 2-4-51 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

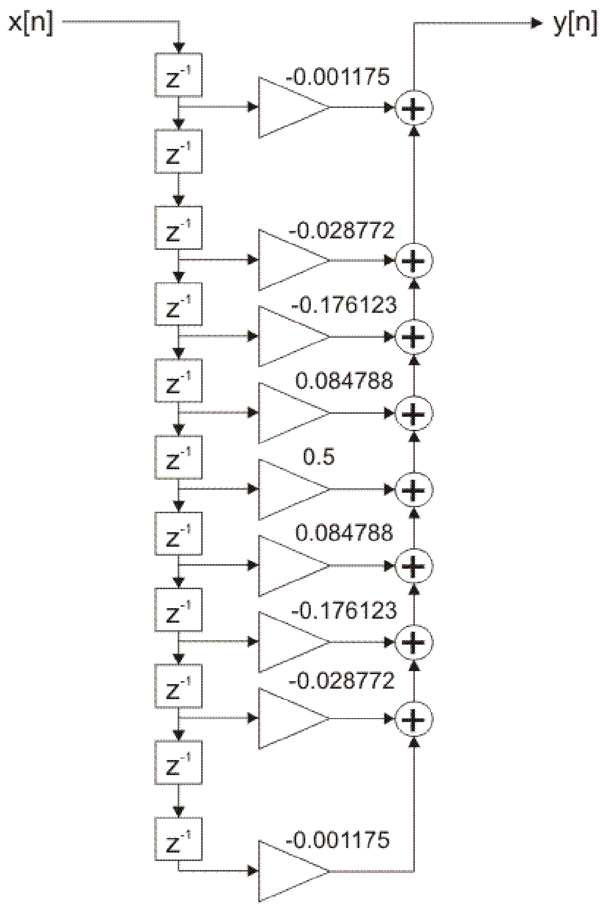


Figure 2-4-50. FIR filter direct realization

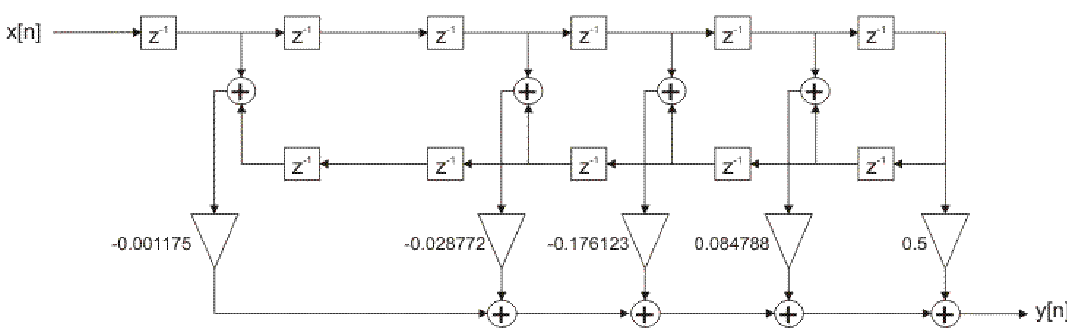


Figure 2-4-51. FIR filter optimized realization structure

2.4.6.4 Example 4

Step 1:

Type of filter – band-stop filter
Filte specifications:

- Filter order – Nf=12;
- Sampling frequency – fs=16000Hz; and
- Passband cut-off frequency – fc1=2KHz, fc2=6KHz.

Step 2:

Method – filter design using Bohman window

Step 3:

Filter order is predetermined, Nf=12;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=13; and
Coefficients have indices between 0 and 12.

Step 4:

The coefficients of Bohman window are found via expression:

$$w[n] = \left(1 - \frac{\left|n - \frac{N-1}{2}\right|}{\frac{N-1}{2}}\right)\cos\left(\pi\frac{\left|n - \frac{N-1}{2}\right|}{\frac{N-1}{2}}\right) + \frac{1}{\pi}\sin\left(\pi\frac{\left|n - \frac{N-1}{2}\right|}{\frac{N-1}{2}}\right); 0 \le n \le N-1$$

w[n]={ 0, 0.014817, 0.108998, 0.31831, 0.608998, 0.880843,
1,
0.880843, 0.608998, 0.31831, 0.108998, 0.014817, 0 }

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \frac{\sin(\omega_{c1}(n-M))}{\pi(n-M)} - \frac{\sin(\omega_{c2}(n-M))}{\pi(n-M)}; & n \ne M \\ 1 - \frac{\omega_{c2} - \omega_{c1}}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N}{2} = 6$$

Normalized cut-off frequencies ωc1 and ωc2 can be calculated using expressions:

$$\omega_{c1} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 2000}{16000} = 0.25\pi$$

$$\omega_{c2} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 6000}{16000} = 0.75\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M, ωc1 and ωc2 with expression for the impulse response coefficients of the ideal band-stop filter:

$h_d[n]$={ -0.106103, 0, 0, 0, 0.31831, 0,
0.5,
0, 0.31831, 0, 0, 0, -0.106103 }

Step 6:

The designed FIR filter coefficients are found via expression:

h[n] = w[n] * hd[n] ; 0 ≤ n ≤ 12

The FIR filter coefficients h[n] rounded to 6 digits are:

h[n]= { 0, 0, 0, 0, 0.19385, 0,
0.5,
0, 0.19385, 0, 0, 0, 0 }

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-52 illustrates the direct realization of designed FIR filter, whereas figure 2-4-53 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

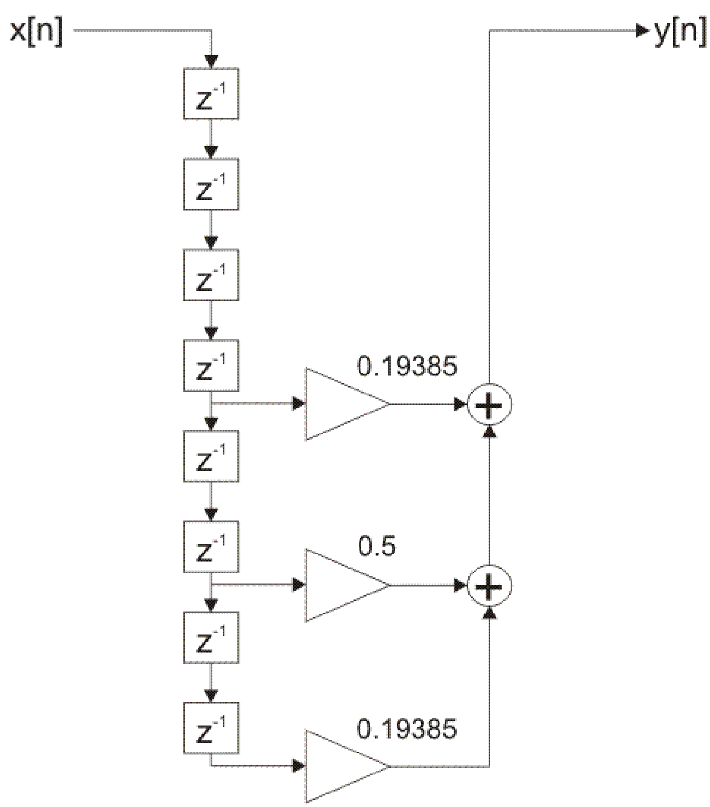


Figure 2-4-52. FIR filter direct realization

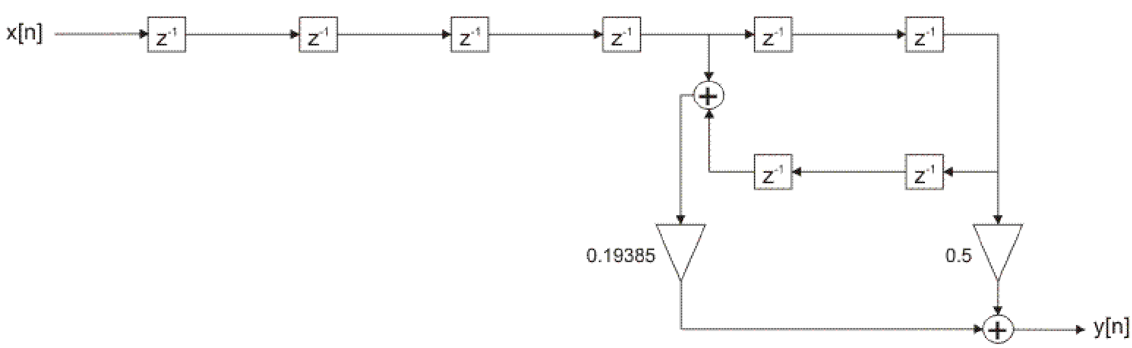


Figure 2-4-53. FIR filter optimized realization structure

2.4.7 Filter design using Blackman window

2.4.7.1 Example 1

Step 1:

Type of filter – low-pass filter
Filte specifications:

- Filter order – Nf=12;
- Sampling frequency – fs=44100KHz;
- Passband cut-off frequency – fc=15KHz; and
- Attenuation of 0dB at 0Hz – 0dB.

Step 2:

Method –Filter design using Blackman window

Step 3:

Filter order is predetermined, Nf=12;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=13;
Coefficients have indices between 0 and 12.

Step 4:

The coefficients of Blackman window are found via expression:

$$w[n] = 0.42 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right); 0 \le n \le N-1$$

w[n]={ 0, 0.026987, 0.13, 0.34, 0.63, 0.893013,
1,
0.893013, 0.63, 0.34, 0.13, 0.026987, 0 }

Step 5:

The ideal low-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \frac{\sin[\omega_c(n-M)]}{\pi(n-M)}; & n \ne M \\ \frac{\omega_c}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N_f}{2} = 6$$

Normalized cut-off frequency ωc can be computed using expression:

$$\omega_c = \frac{2\pi f_c}{f_s} = \frac{2\cdot\pi\cdot 15000}{44100} = 0.6803\pi$$

The values of coefficients are obtained (rounded to six digits) by combining the values of M and ωc with expression for the impulse response coefficients of the ideal low-pass filter:

h_d[n]={ 0.013457, -0.06063, 0.061142, 0.013568, -0.144123, 0.268612,
0.680272,
0.268612, -0.144123, 0.013568, 0.061142, -0.06063, 0.013457 }

Step 6:

The designed FIR filter coefficients are found via expression:

h[n] = w[n] * hd[n] ; 0 ≤ n ≤ 12

The FIR filter coefficients h[n] rounded to 6 digits are:

h[n]={ 0, -0.001636, 0.007948, 0.004613, -0.090798, 0.239874,
0.680272,
0.239874, -0.090798, 0.004613, 0.007948, -0.001636, 0 }

The resulting coefficients must be scaled in order to provide attenuation of 0dB at 0Hz. In order to provide attenuation of 0 dB, the following condition must be met:

$$\sum_{n=0}^{N-1} h[n] = 1$$

The sum of the previously obtained coefficients is:

$$\sum_{n=0}^{12} h[n] = 1.000274$$

As the sum is greater than one, it is necessary to divide all coefficients of the impulse response by 1.000274. After division, these coefficients have the following values:

$$h[n]=\{\ 0,\ -0.001636,\ 0.007946,\ 0.004612,\ -0.090773,\ 0.239808,\ 0.680086,\ 0.239808,\ -0.090773,\ 0.004612,\ 0.007946,\ -0.001636,\ 0\ \}$$

The sum of scaled coefficients is equal to 1, which means that attenuation at 0Hz frequency amounts to 0dB. Note that these coefficients cannot be used in designing a FIR filter safe from filtering overflow. In order to prevent a filtering overflow from occuring it is necessary to satisfy the condition below:

$$\sum_{n=0}^{N-1} |h[n]| \le 1$$

The resulting filter doesn't meet this condition. Negative coefficients in impulse response make that both conditions cannot be met. The sum of apsolute values of coefficients in the resulting filter is:

$$\sum_{n=0}^{12} |h[n]| = 1.369636$$

The sum of coefficients apsolute values before scaling amounts to 1.37001 (1.369636·1.000274). After scaling, it is somewhat less, so it is less likely that an overflow occurs. In such cases, possible filtering overflows are not dangerous. Namely, most processors containing hardware multipliers (which is almost necessary for filtering) have registers with extended band. In this case, it is far more important to faithfully transmit a direct signal to a FIR filter output.

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-54 illustrates the direct realization of designed FIR filter, whereas figure 2-4-55 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

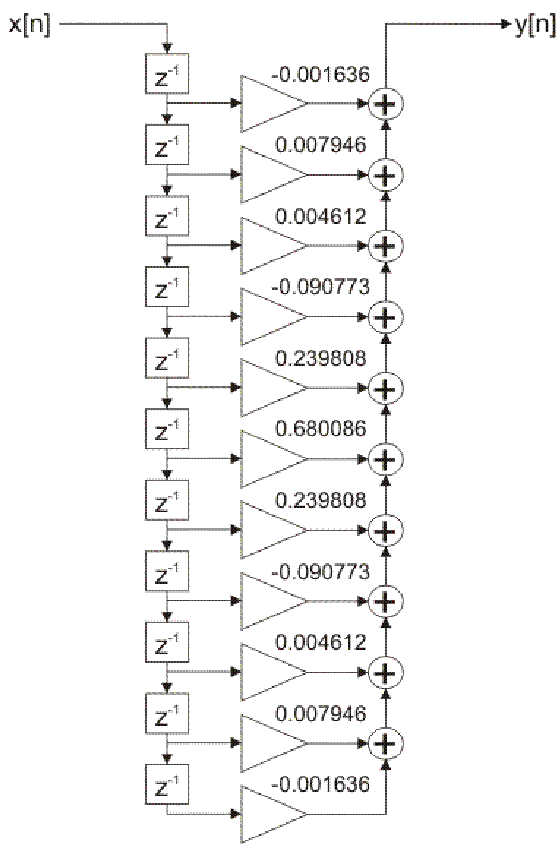


Figure 2-4-54. FIR filter direct realization

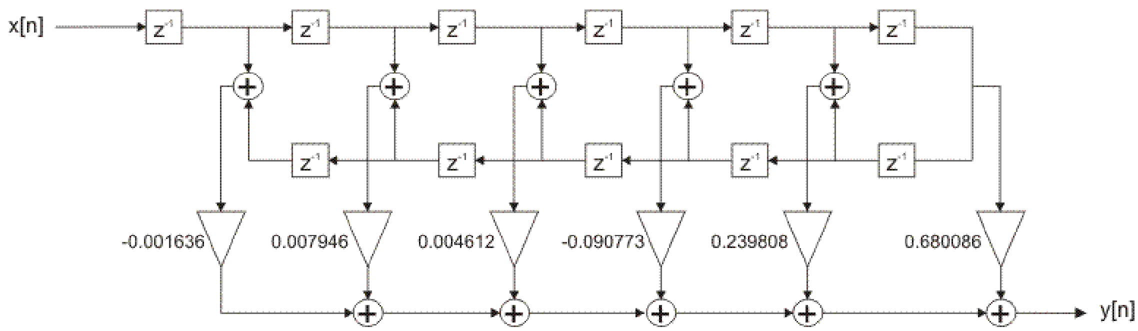


Figure 2-4-55. FIR filter optimized realization structure

2.4.7.2 Example 2

Step 1:

Type of filter – high-pass filter
Filte specifications:

- Filter order– Nf=12;
- Sampling frequency – fs=22050Hz;
- Passband cut-off frequency – fc=4KHz;
- Prevention of possible filtering overflows.

Step 2:

Method –filter design using Blackman window

Step 3:

Filter order is Nf=12;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=13;
Coefficients have indices between 0 and 12.

Step 4:

The coefficients of Blackman window function are found via:

$$w[n] = 0.42 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right); 0 \le n \le N-1$$

$$w[n]=\{\ 0,\ 0.026987,\ 0.13,\ 0.34,\ 0.63,\ 0.893013,\ 1,\ 0.893013,\ 0.63,\ 0.34,\ 0.13,\ 0.026987,\ 0\ \}$$

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} 1-\dfrac{\omega_c}{\pi}; & n \neq M \\ -\dfrac{\sin(\omega_c(n-M))}{\pi(n-M)}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N_f}{2} = 6$$

Normalized cut-off frequency ωc can be computed using expression:

$$\omega_c = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 4000}{22050} = 0.3628\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc with expression for the impulse response coefficients of the ideal high-pass filter:

$$h_d[n]=\{ -0.027985, 0.035109, 0.078646, 0.029101, -0.12082, -0.289201, 0.637188, -0.289201, -0.12082, 0.029101, 0.078646, 0.035109, -0.027985 \}$$

Step 6:

The designed FIR filter coefficients are found via expression:

h[n] = w[n] * hd[n] ; 0 ≤ n ≤ 12

The FIR filter coefficients h[n] rounded to 6 digits are:

$$h[n]=\{ 0, 0.000947, 0.010224, 0.009894, -0.076116, -0.25826, 0.637188, -0.25826, -0.076116, 0.009894, 0.010224, 0.000947, 0 \}$$

In order to prevent filtering overflow, the following condition must be met:

$$\sum_{n=0}^{N-1} |h[n]| = 1$$

The sum of absolute values of the resulting FIR filter coefficients is:

$$\sum_{n=0}^{12} |h[n]| = 1.34807$$

The obtained coefficients must be scaled (divided) by 1.34807. After that, their values are:

$$h[n]=\{ 0, 0.000702, 0.007584, 0.007339, -0.056463, -0.191578, 0.472667, -0.191578, -0.056463, 0.007339, 0.007584, 0.000702, 0 \}$$

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-56 illustrates the direct realization of designed FIR filter, whereas figure 2-4-57 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

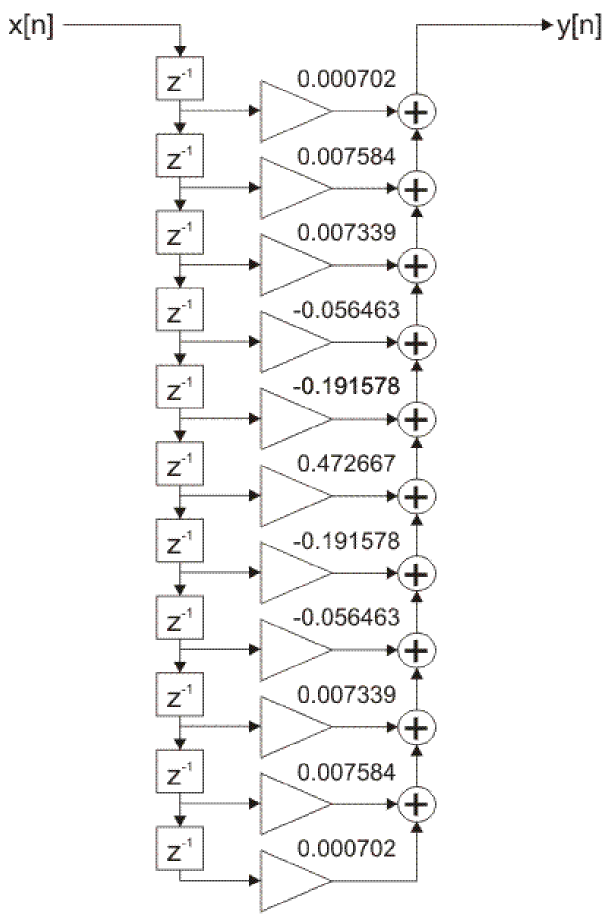


Figure 2-4-56. FIR filter direct realization

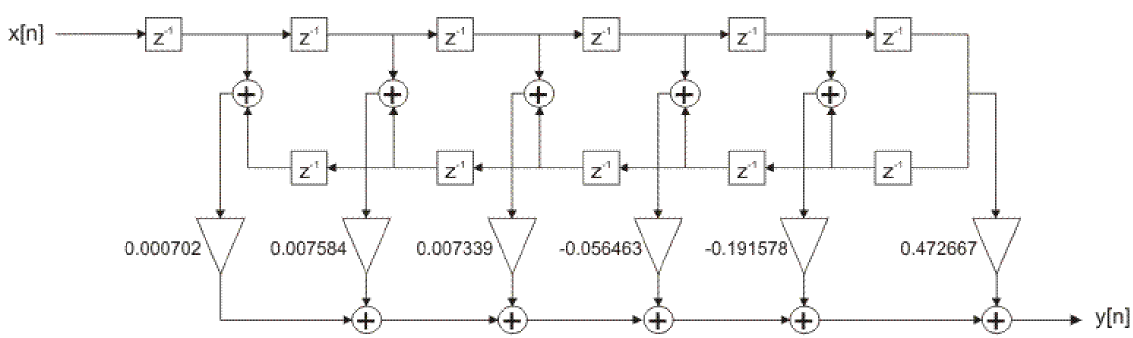


Figure 2-4-57. FIR filter optimized realization structure

2.4.7.3 Example 3

Step 1:

Type of filter – band-pass filter
Filte specifications:

- Filter order – Nf=12;
- Sampling frequency – fs=44100Hz;
- Passband cut-off frequency – fc1=4KHz, fc2=15025Hz;
- Prevention of possible filtering overflow.

Step 2:

Method –filter design using Blackman window

Step 3:

Filter order is predetermined, Nf=12;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=13;
Coefficients have indices between 0 and 12.

Step 4:

The coefficients of Blackman window function are found via expression:

$$w[n] = 0.42 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right); 0 \le n \le N-1$$

$$w[n]=\{ 0, 0.026987, 0.13, 0.34, 0.63, 0.893013, 1, 0.893013, 0.63, 0.34, 0.13, 0.026987, 0 \}$$

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \frac{\sin(\omega_{c2}(n-M))}{\pi(n-M)} - \frac{\sin(\omega_{c1}(n-M))}{\pi(n-M)}; & n \ne M \\ \frac{\omega_{c2} - \omega_{c1}}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N}{2} = 6$$

Normalized cut-off frequencies ωc1 and ωc2 can be computed using expressions:

$$\omega_{c1} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 4000}{44100} = 0.1814\pi$$

$$\omega_{c2} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 15025}{44100} = 0.6814\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M, ωc1 and ωc2 with expression for the impulse response coefficients of the ideal band-pass filter:

$$h_d[n]=\{ 0.029101, -0.079297, 0, -0.090389, -0.289201, 0.096258, 0.5, 0.096258, -0.289201, -0.090389, 0, -0.079297, 0.029101 \}$$

Step 6:

The designed FIR filter coefficients are found via expression:

h[n] = w[n] * hd[n] ; 0 ≤ n ≤ 12

The FIR filter coefficients h[n] rounded to 6 digits are:

$$h[n]= \{ 0, -0.00214, 0, -0.030732, -0.182197, 0.085959, 0.5, 0.085959, -0.182197, -0.030732, 0, -0.002140, 0 \}$$

In order to prevent filtering overflows, the following condition must be met:

$$\sum_{n=0}^{N-1} |h[n]| = 1$$

The sum of absolute values of the resulting FIR filter coefficients is:

$$\sum_{n=0}^{12} |h[n]| = 1.102056$$

The obtained coefficients must be scaled (divided) by 1.102056. After this, their values are:

$$h[n] = \{ 0, -0.001942, 0, -0.027886, -0.165325, 0.077999, 0.453697, 0.077999, -0.165325, -0.027886, 0, -0.001942, 0 \}$$

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-58 illustrates the direct realization of designed FIR filter, whereas figure 2-4-59 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

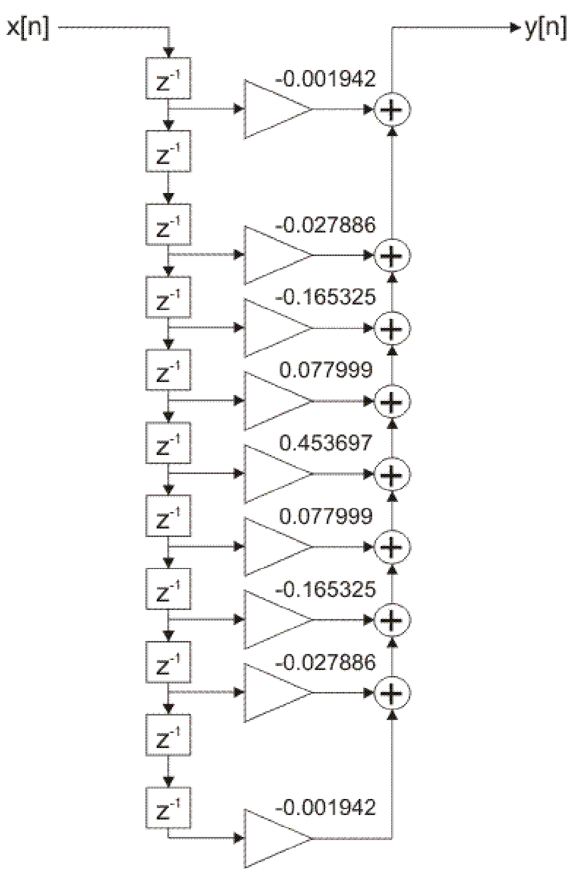


Figure 2-4-58. FIR filter direct realization

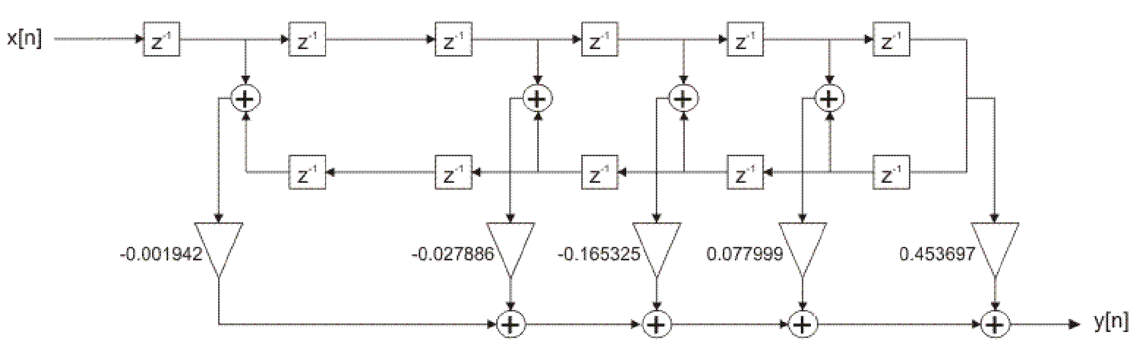


Figure 2-4-59. FIR filter optimized realization structure

2.4.7.4 Example 4

Step 1:

Type of filter – band-stop filter
Filte specifications:

- Filter order – Nf=12;
- Sampling frequency – fs=16000Hz;
- Passband cut-off frequencies – fc1=2KHz, fc2=6KHz;
- Prevention of possible filtering overflows.

Step 2:

Method – filter design using Blackman window

Step 3:

Filter order is predetermined, Nf=12;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=13;
Coefficients have indices between 0 and 12.

Step 4:

The coefficients of Blackman window are found via expression:

$$w[n] = 0.42 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right); 0 \le n \le N-1$$

$$w[n]=\{\ 0,\ 0.026987,\ 0.13,\ 0.34,\ 0.63,\ 0.893013,\ 1,\ 0.893013,\ 0.63,\ 0.34,\ 0.13,\ 0.026987,\ 0\ \}$$

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \dfrac{\sin(\omega_{c1}(n-M))}{\pi(n-M)} - \dfrac{\sin(\omega_{c2}(n-M))}{\pi(n-M)}; & n \ne M \\ 1 - \dfrac{\omega_{c2} - \omega_{c1}}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N}{2} = 6$$

Normalized cut-off frequencies ωc1 and ωc2 can be computed using expressions:

$$\omega_{c1} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 2000}{16000} = 0.25\pi$$

$$\omega_{c2} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 6000}{16000} = 0.75\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M, ωc1 and ωc2 with expression for the impulse response coefficients of the ideal band-stop filter:

$$h_d[n]=\{\ -0.106103,\ 0,\ 0,\ 0,\ 0.31831,\ 0,\ 0.5,\ 0,\ 0.31831,\ 0,\ 0,\ 0,\ -0.106103\ \}$$

Step 6:

The designed FIR filter coefficients are found via expression:

h[n] = w[n] * hd[n] ; 0 ≤ n ≤ 12

The FIR filter coefficients h[n] rounded to 6 digits are:

$$h[n] = \{ 0, 0, 0, 0, 0.200535, 0, 0.5, 0, 0.200535, 0, 0, 0, 0 \}$$

In order to prevent filtering overflows, the following condition must be met:

$$\sum_{n=0}^{N-1} |h[n]| = 1$$

The sum of absolute values of the resulting FIR filter coefficients is:

$$\sum_{n=0}^{12} |h[n]| = 0.90107$$

The obtained coefficients must be scaled (divided) by 0.90107. After this, their values are:

$$h[n] = \{ 0, 0, 0, 0, 0.222552, 0, 0.554896, 0, 0.222552, 0, 0, 0, 0 \}$$

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-60 illustrates the direct realization of designed FIR filter, whereas figure 2-4-61 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

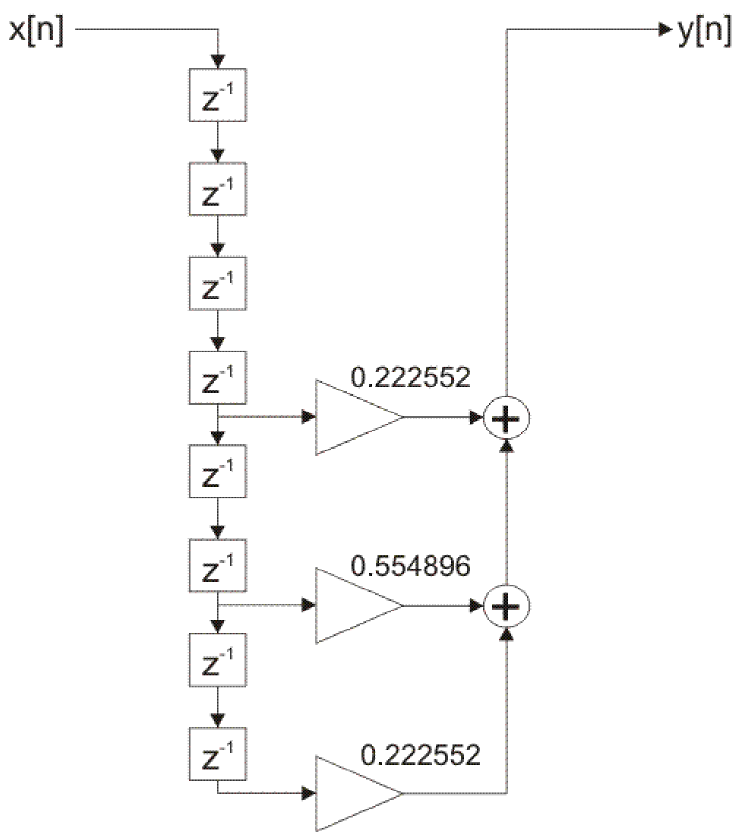


Figure 2-4-60. FIR filter direct realization

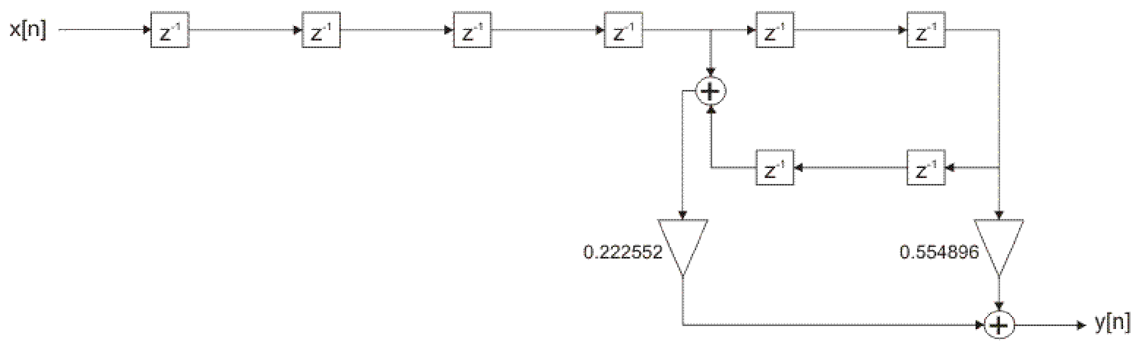


Figure 2-4-61. FIR filter optimized realization structure

2.4.8 Filter design using Blackman-Harris window

2.4.8.1 Example 1

Step 1:

Type of filter –low-pass filter
Filte specifications:

- Filter order – Nf=12;
- Sampling frequency – fs=44100KHz;
- Passband cut-off frequency – fc=15KHz;
- Attenuation of 0dB at 0Hz.

Step 2:

Method –filter design usng Blackman-Harris window

Step 3:

Filter order is predetermined, Nf=12;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=13; and
Coefficients have indices between 0 and 12.

Step 4:

The coefficients of Blackman-Harris window are found via expression:

$$w[n] = 0,35875 - 0,48829\cos\left(\frac{2\pi n}{N-1}\right) + 0,14128\cos\left(\frac{4\pi n}{N-1}\right) - 0,01168\cos\left(\frac{6\pi n}{N-1}\right); 0 \le n \le N-1$$

w[n]={ 0.00006, 0.065478, 0.244205, 0.48835, 0.732495, 0.911222,
0.97664,
0.911222, 0.732495, 0.48835, 0.244205, 0.065478, 0.00006 }

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \dfrac{\sin[\omega_c(n-M)]}{\pi(n-M)}; & n \ne M \\ \dfrac{\omega_c}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N_f}{2} = 6$$

Normalized cut-off frequency ωc may be calculated using expression:

$$\omega_c = \frac{2\pi f_c}{f_s} = \frac{2\cdot\pi\cdot 15000}{44100} = 0.6803\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc with expression for the impulse response coefficients of the ideal low-pass filter:

$$h_d[n]=\{ 0.013457, -0.06063, 0.061142, 0.013568, -0.144123, 0.268612, 0.680272, 0.268612, -0.144123, 0.013568, 0.061142, -0.06063, 0.013457 \}$$

Step 6:

The designed FIR filter coefficients are found via expression:

$$h[n] = w[n] * hd[n] ; 0 \le n \le 12$$

The FIR filter coefficients h[n] rounded to 6 digits are:

$$h[n]=\{ 0.000001, -0.00397, 0.014931, 0.006626, -0.10557, 0.244765, 0.664381, 0.244765, -0.10557, 0.006626, 0.014931, -0.00397, 0.000001 \}$$

The resulting coefficients must be scaled in order to provide attenuation of 0dB at 0Hz. To provide 0 dB attenuation, the following condition must be met:

$$\sum_{n=0}^{N-1} h[n] = 1$$

The sum of the previously obtained coefficients is:

$$\sum_{n=0}^{12} h[n] = 0.977947$$

As the sum is greater than one, it is necessary to divide all the impulse response coefficients by 0.977947. After this, the values of these coefficients are:

$$h[n]=\{ 0.000001, -0.00406, 0.015268, 0.006775, -0.107951, 0.250285, 0.679363, 0.250285, -0.107951, 0.006775, 0.015268, -0.00406, 0.000001 \}$$

The sum of scaled coefficients is equal to 1, which means that attenuation at 0Hz frequency amounts to 0dB. Note that these coefficients cannot be used in designing a FIR filter safe from filtering overflow. In order to prevent a filtering overflow from occuring it is necessary to satisfy the condition below:

$$\sum_{n=0}^{N-1} |h[n]| \le 1$$

The resulting filter doesn’t meet this condition. Negative coefficients in impulse response indicate that both conditions cannot be met. The sum of apsolute values of coefficients in the resulting filter is:

$$\sum_{n=0}^{12} |h[n]| = 1.448043$$

The sum of coefficients apsolute values before scaling amounts to 1.37001 (1.369636·1.000274). After scaling, the sum of coefficients apsolute values is somewhat less, so it is less possible that an overflow occurs. In such cases, possible filtering overflows are not dangerous. Namely, most processors containing hardware multipliers (which is almost necessary for filtering) have registers with extended band. In this case, it is far more important to faithfully transmit a direct signal to a FIR filter output.

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-62 illustrates the direct realization of designed FIR filter, whereas figure 2-4-63 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

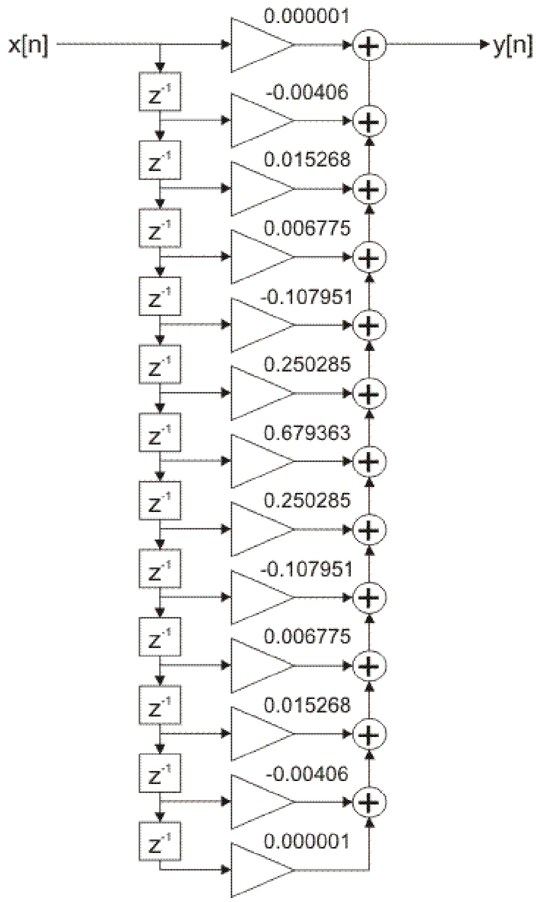


Figure 2-4-62. FIR filter

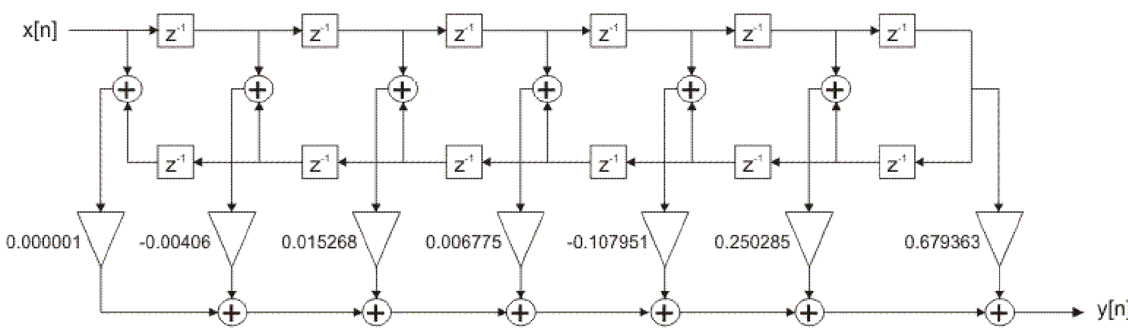


Figure 2-4-63. Optimized FIR filter design

2.4.8.2 Example 2

Step 1:

Type of filter – high-pass filter
Filte specifications:

- Filter order – Nf=12;
- Sampling frequency – fs=22050Hz;
- Passband cut-off frequency – fc=4KHz;
- Prevention of filtering overflows.

Step 2:

Method – filter design using Blackman-Harris window

Step 3:

Filter order is Nf=12;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=13; and
Coefficients have indices between 0 and 12.

Step 4:

The coefficients of Blackman-Harris window are found via:

$$w[n] = 0{,}35875 - 0{,}48829\cos\left(\frac{2\pi n}{N-1}\right) + 0{,}14128\cos\left(\frac{4\pi n}{N-1}\right) - 0{,}01168\cos\left(\frac{6\pi n}{N-1}\right); 0 \le n \le N-1$$

w[n]={ 0.00006, 0.065478, 0.244205, 0.48835, 0.732495, 0.911222,
0.97664,
0.911222, 0.732495, 0.48835, 0.244205, 0.065478, 0.00006 }

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} 1 - \frac{\omega_c}{\pi}; & n \ne M \\ -\frac{\sin(\omega_c(n-M))}{\pi(n-M)}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N_f}{2} = 6$$

Normalized cut-off frequency ωc can be computed using expression:

$$\omega_c = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 4000}{22050} = 0.3628\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M and ωc with the expression for the impulse response coefficients of the ideal high-pass filter:

h_d[n]={ -0.027985, 0.035109, 0.078646, 0.029101, -0.12082, -0.289201,
0.637188,
-0.289201, -0.12082, 0.029101, 0.078646, 0.035109, -0.027985 }

Step 6:

The designed FIR filter coefficients are found via expression:

h[n] = w[n] * hd[n] ; 0 ≤ n ≤ 12

The FIR filter coefficients h[n] rounded to 6 digits are:

h[n]={ -0.000002, 0.002299, 0.019206, 0.014211, -0.0885, -0.263526,
0.622303,
-0.263526, -0.0885, 0.014211, 0.019206, 0.002299, -0.000002 }

In order to prevent filtering overflow, the following condition must be met:

$$\sum_{n=0}^{N-1} |h[n]| = 1$$

The sum of absolute values of the resulting FIR filter coefficients is:

$$\sum_{n=0}^{12} |h[n]| = 1.397791$$

The obtained coefficients must be scaled (divided) by 1.397791. After this, their values are:

h[n]={ -0.000001, 0.001645, 0.01374, 0.010167, -0.063314, -0.18853,
0.445205,
-0.18853, -0.063314, 0.010167, 0.01374, 0.001645, -0.000001 }

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-64 illustrates the direct realization of designed FIR filter, whereas figure 2-4-65 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

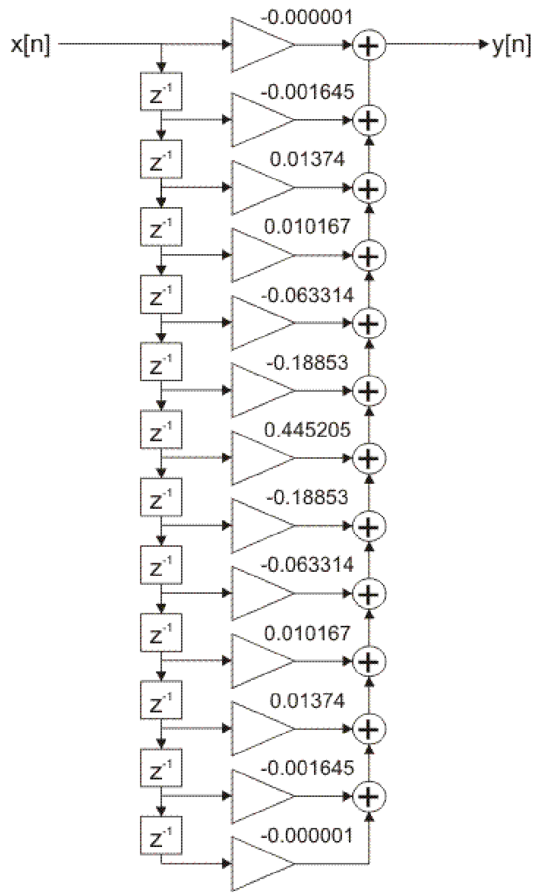


Figure 2-4-64. FIR filter direct realization

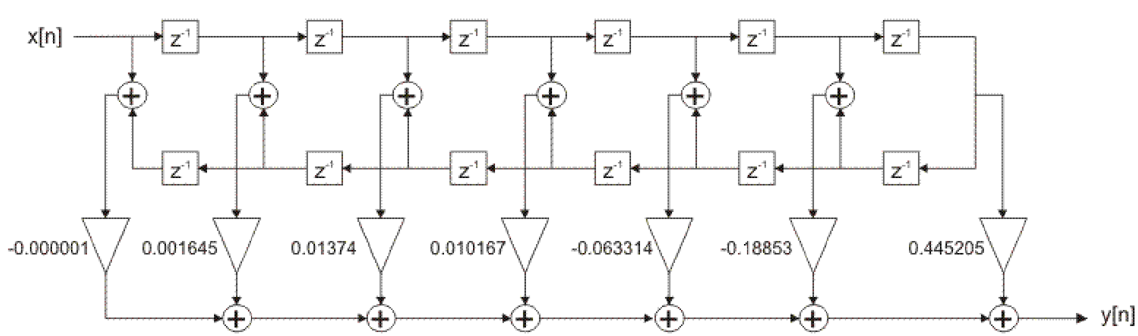


Figure 2-4-65. Optimized FIR filter design

2.4.8.3 Example 3

Step 1:

Type of filter –band-pass filter
Filter specification:

- Filter order – Nf=12;
- Sampling frequency – fs=44100Hz;
- Passband cut-off frequencies – fc1=4KHz, fc2=15025Hz; and
- Prevention of possible filtering overflows.

Step 2:

Method – filter design using Blackman-Harris window

Step 3:

Filter order is predetermined, Nf=12;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=13; and
Coefficients have indices between 0 and 12.

Step 4:

The coefficients of Blackman-Harris window are found via expression:

$$w[n] = 0{,}35875 - 0{,}48829\cos\left(\frac{2\pi n}{N-1}\right) + 0{,}14128\cos\left(\frac{4\pi n}{N-1}\right) - 0{,}01168\cos\left(\frac{6\pi n}{N-1}\right); 0 \le n \le N-1$$

w[n]={ 0.00006, 0.065478, 0.244205, 0.48835, 0.732495, 0.911222,
0.97664,
0.911222, 0.732495, 0.48835, 0.244205, 0.065478, 0.00006 }

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \dfrac{\sin(\omega_{c2}(n-M))}{\pi(n-M)} - \dfrac{\sin(\omega_{c1}(n-M))}{\pi(n-M)}; & n \ne M \\ \dfrac{\omega_{c2} - \omega_{c1}}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N}{2} = 6$$

Normalized cut-off frequencies ωc1 and ωc2 can be computed using expressions:

$$\omega_{c1} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 4000}{44100} = 0.1814\pi$$

$$\omega_{c2} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 15025}{44100} = 0.6814\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M, ωc1 and ωc2 with expression for the impulse response coefficients of the ideal band-pass filter:

h_d[n]={ 0.029101, -0.079297, 0, -0.090389, -0.289201, 0.096258,
0.5,
0.096258, -0.289201, -0.090389, 0, -0.079297, 0.029101 }

Step 6:

The designed FIR filter coefficients are found via expression:

h[n] = w[n] * hd[n] ; 0 ≤ n ≤ 12

The FIR filter coefficients h[n] rounded to 6 digits are:

h[n]= { 0.000002, -0.005192, 0, -0.044142, -0.211839, 0.087712,
0.48832,
0.087712, -0.211839, -0.044142, 0, -0.005192, 0.000002 }

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter reaization:

Figure 2-4-66 illustrates the direct realization of designed FIR filter, whereas figure 2-4-67 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

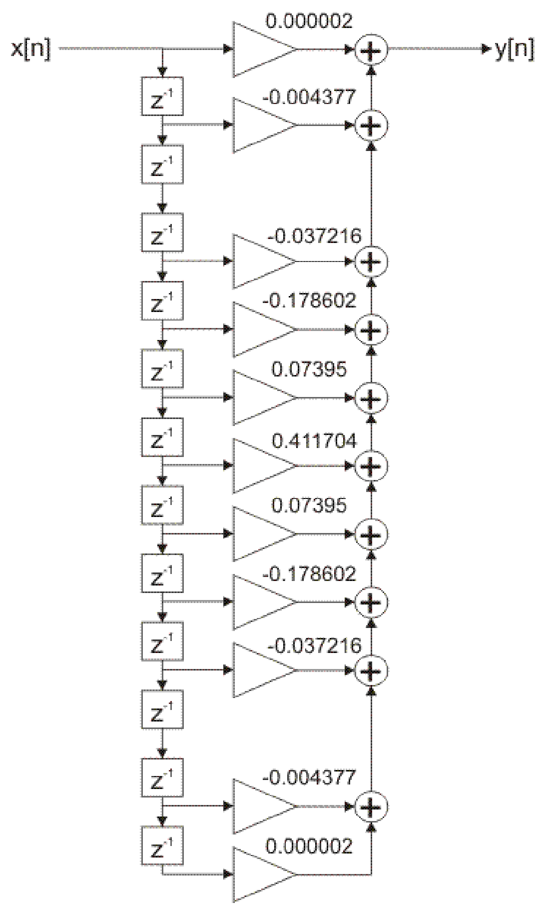


Figure 2-4-66. FIR filter direct realization

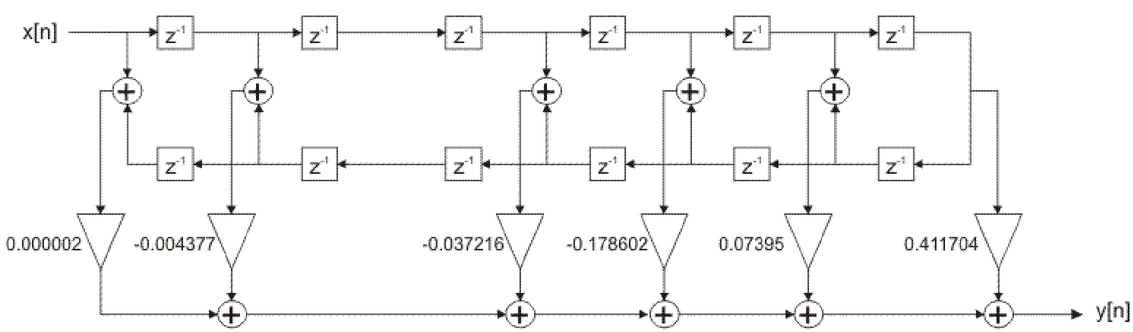


Figure 2-4-67. Optimized FIR filter design

2.4.8.4 Example 4

Step 1:

Type of filter – band-stop filter
Filter specification:

- Filter order – Nf=20;
- Sampling frequency – fs=16000Hz;
- Passband cut-off frequency – fc1=2KHz, fc2=6KHz; and
- Prevention of possible filtering overflows.

Step 2:

Method – filter design using Blackman-Harris window

Step 3:

Filter order is predetermined, Nf=20;
A total number of filter coefficients is larger by 1, i.e. N=Nf+1=21; and
Coefficients have indices between 0 and 20.

Step 4:

The coefficients of Blackman-Harris window are found via expression:

$$w[n] = 0{,}35875 - 0{,}48829\cos\left(\frac{2\pi n}{N-1}\right) + 0{,}14128\cos\left(\frac{4\pi n}{N-1}\right) - 0{,}01168\cos\left(\frac{6\pi n}{N-1}\right); 0 \le n \le N-1$$

w[n]={ 0.00006, 0.023959, 0.093315, 0.201340, 0.33746,
0.48835, 0.63924, 0.77536, 0.883385, 0.952741,
0.97664,
0.952741, 0.883385, 0.77536, 0.63924, 0.48835,
0.33746, 0.20134, 0.093315, 0.023959, 0.00006 }

Step 5:

The ideal high-pass filter coefficients (ideal filter impulse response) are expressed as:

$$h_d[n] = \begin{cases} \dfrac{\sin(\omega_{c1}(n-M))}{\pi(n-M)} - \dfrac{\sin(\omega_{c2}(n-M))}{\pi(n-M)}; & n \neq M \\ 1 - \dfrac{\omega_{c2} - \omega_{c1}}{\pi}; & n = M \end{cases}$$

where M is the index of middle coefficient.

$$M = \frac{N}{2} = 10$$

Normalized cut-off frequencies ωc1 and ωc2 can be computed using expressions:

$$\omega_{c1} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 2000}{16000} = 0.25\pi$$

$$\omega_{c2} = \frac{2\pi f_c}{f_s} = \frac{2 \cdot \pi \cdot 6000}{16000} = 0.75\pi$$

The values of coefficients (rounded to six digits) are obtained by combining the values of M, ωc1 and ωc2 with expression for the impulse response coefficients of the ideal band-stop filter:

$h_d[n]$={ 0.063662, 0, 0, 0, -0.106103, 0, 0, 0, 0.31831, 0,
0.5,
0, 0.31831, 0, 0, 0, -0.106103, 0, 0, 0, 0.063662 }

Step 6:

The designed FIR filter coefficients are found via expression:

h[n] = w[n] * hd[n] ; 0 ≤ n ≤ 20

The FIR filter coefficients h[n] rounded to 6 digits are:

h[n]= { 0.000004, 0, 0, 0, -0.035806, 0, 0, 0, 0.28119, 0,
0.48832,
0, 0.28119, 0, 0, 0, -0.035806, 0, 0, 0, 0.000004 }

In order to prevent filtering overflows, the following condition must be met:

$$\sum_{n=0}^{N-1} |h[n]| = 1$$

The sum of absolute values of the resulting FIR filter coefficients is:

$$\sum_{n=0}^{12} |h[n]| = 1.12232$$

The obtained coefficients must be scaled (divided) by 1.12232. After this, their values are:

$$h[n]= \{ 0.000004, 0, 0, 0, -0.031904, 0, 0, 0, 0.250544, 0, 0.435099, 0, 0.250544, 0, 0, 0, -0.031904, 0, 0, 0, 0.000004 \}$$

Step 7:

The filter order is predetermined.
There is no need to additionally change it.

Filter realization:

Figure 2-4-68 illustrates the direct realization of designed FIR filter, whereas figure 2-4-69 illustrates optimized realization structure of designed FIR filter which is based on the fact that all FIR filter coefficients are, for the sake of linear phase characteristic, symmetric about their middle element.

This FIR filter is an excellent example showing the importance of the sampling frequency. It is specified to give the passband amounting to 0.5π. This causes most impulse response coefficients of the resulting FIR filter to be zeros. It further makes the filter realization structure simpler. As for optimized FIR filter design, there are only 4 multiplications, even though the filter is of 20th order. Unfortunatelly, the buffer length cannot be minimized. It is fixed and corresponds to the filter order. However, it is possible to affect design complexity, whether it is hardware or software implementation.

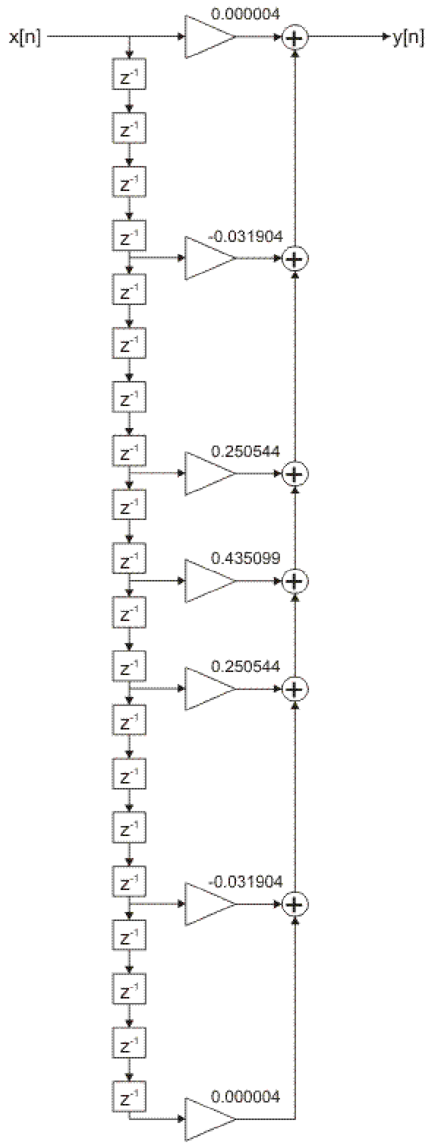


Figure 2-4-68. FIR filter direct realization

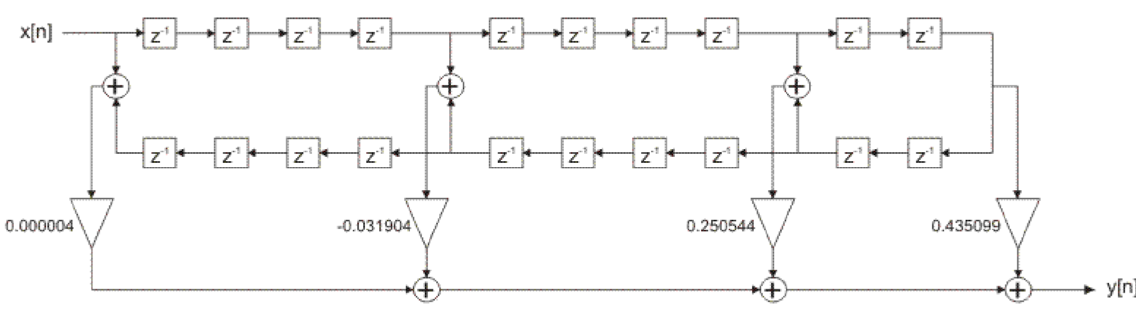


Figure 2-4-69. Optimized FIR filter design

2.5 Finite word-length effects

There are hardware and software FIR filter realizations. Regardless of which of them is used, a problem known as the finite word-length effect exists in either case. One of the objectives, when designing filters, is to lessen the finite word-length effects as much as possible, thus satisfying the initiative requirements (filter specifications).

On software filter implementation, it is possible to use either fixed-point or floating-point arithmetic. Both representations of numbers have some advantages and disadvantages as well.

The fixed-point representation is used for saving coefficients and samples in memory. Most commonly used fixed-point format is when one bit denotes a sign of a number, i.e. 0 denotes a positive, whereas 1 denotes a negative number, and the rest of bits denote the value of a number. This is mostly used to represent numbers in the range -1 to +1. Numbers represented in the fixed-point format are equidistantly quantized with the quantization step 1/2N-1, where N is the number of a bit used for saving the value. As one bit is a sign bit, there are N-1 bits available for value quantization. The maximum error that may occur during quantization is 1/2 quantization step, that is 1/2N. It can be noted that accuracy increases as the number of bits increases. Table 2-5-1 shows the values of quantization steps and maximum errors made due to quantization process in the fixed-point presentation.

BIT NUMBER	RANGE OF NUMBERS	QUANTIZATION STEP	MAX. QUANTIZATION ERROR	NUMBER OF EXACT DECIMAL POINTS
4	(-1, +1)	0.125	0.0625	1
8	(-1, +1)	0.0078125	0.00390625	2
16	(-1, +1)	3.0517578125*10-5	1.52587890625*10-5	4
32	(-1, +1)	4.6566128730774*10-10	2.3283064365387*10-10	9
64	(-1, +1)	1.0842021724855*10-19	5.4210108624275*10-20	19

Table 2-5-1. Quantization of numbers represented in the fixed-point format

The advantage of this presentation is that quantization errors tend to approximate 0. It means that errors are not accumulated in operations performed upon fixed-point numbers. One of disadvantages is a smaller accuracy in coefficients representation. The difference between actual sampled value and quantized value, i.e. the quantization error, is smaller as the quantization level decreases. In other words, the effects of the quantization error are negligible in this case.

The floating-point arithmetic saves values with better accuracy due to dynamics it is based on. Floating-point representations cover a much wider range of numbers. It also enables an appropriate number of digits to be faithfully saved. The value normally consists of three parts. The first part is, similar to the fixed-point format, represented by one bit known as the sign bit. The second part is a mantissa M, which is a fractional part of the number, and the third part is an exponent E, which can be either positive or negative. A number in the floating-point format looks as follows:

$$Val = \pm M \cdot 10^E$$

where M is the mantissa and E is the exponent.

As seen, the sign bit along with mantissa represent a fixed-point format. The third part, i.e. exponent provides the floating-point representation with dynamics, which further enables both extremely large and extremely small numbers to be saved with appropriate accuracy. Such numbers could not be represented in the fixed-point format. Table 2-5-2 below provides the basic information on floating-point representation for several different lengths.

BIT NUMBER	MANTISSA SIZE	EXPONENT SIZE	BAND	NUMBER OF EXACT DECIMAL POINTS
16	7	8	2.3x10-38 .. 3.4x1038	2
32	23	8	1.4x10-45 .. 3.4x1038	6-7

Table 2-5-2. Quantization of numbers represented in the floating-point format

It is not possible to determine the quantization step in the floating-point representation as it depends on exponent. Exponent varies in a way that the quantization step is as small as possible. In this number presentation, special attention should be paid to the number of digits that are saved with no error.

The floating-point arithmetic is suitable for coefficient representation. The errors made in this case are considerably less than those made in the fixed-point arithmetic. Some of disadvantages of this presentation are complex implementation and errors that do not tend to approximate 0. The problem is extremely obvious when the operation is performed upon two values of which one is much less than the other.

Example

FIR filter coefficients:

{0.151365, 0.400000, 0.151365}

Coefficients need to be represented as 16-bit numbers in the fixed-point and floating-point formats. If we suppose that numbers range between -1 and +1, then quantization level amounts to 1 / 2^16 = 0.0000152587890625. After quantization, the filter coefficients have the following values:

{0.1513671875, 0.399993896484375, 0.1513671875}

Quantization errors are:

{-0.0000021875, 0.000006103515625, -0.0000021875}

If filter coefficients are represented in the floated-point format, it is not possible to determine quantization level. In this case, the coefficients have the following values:

{0.151364997029305, 0.400000005960464, 0.151364997029305}

Quantization errors produced while representing coefficients as 16-bit numbers in the floating-point format are:

{0.000000002970695, -0.000000005960464, 0.000000002970695}

As seen, a coefficient error is less in the floating-point representation.

Floating-point arithmetic can also be expressed in terms of fixed-point arithmetic. For this reason, the fixed-point arithmetic is more often implemented in digital signal processors.

The finite word-length effect is the deviation of FIR filter characteristic. If such characteristic still meets the filter specifications, the finite word-length effects are negligible.

As a result of greater error in coefficients representation, the finite word-length effects are more prominent in fixed-point arithmetic.

These effects are more prominent for IIR filters for their feedback property than for FIR filters. In addition, coefficient representation can cause IIR filters to become instable, whereas it cannot affect FIR filters that way.

FIR filters keep their linear phase characteristic after quantization. The reason for this is the fact that the coefficients of a FIR filter with linear phase characteristic are symmetric, which means that the corresponding pairs of coefficients will be quantized to the same value. It results in the impulse response symmetry remaining unchanged.

After all mentioned, it is easy to notice that finite word length, used for representing coefficients and samples being processed, causes some problems such as:

1. Coefficient quantization errors;
2. Sample quantization errors (quantization noise); and
3. Overflow errors.

2.5.1 Coefficient Quantization

The coefficient quantization results in FIR filter changing its transform function. The position of FIR filter zeros is also changed, whereas the position of its poles remains unchanged as they are located in z=0. Quantization has no effect on them. The conclusion is that quantization of FIR filter coefficients cannot cause a filter to become instable as is the case with IIR filters.

Even though there is no danger of FIR filter destabilization, it may happen that transfer function is deviated to such an extent that it no longer meets the specifications, which further means that the resulting filter is not suitable for intended implementation.

The FIR filter quantization errors cause the stopband attenuation to become lower. If it drops below the limit defined by the specifications, the resulting filter is useless.

Transfer function changes occurring due to FIR filter coefficient quantization are more effective for high-order filters. The reason for this is the fact that spacing between zeros of the transfer function get smaller as the filter order increases and such slight changes of zero positions affect the FIR filter frequency response.

2.5.2 Samples Quantization

Another problem caused by the finite word length is sample quantization performed at multiplier's output (after filtering). The process of filtering can be represented as a sum of multiplications performed upon filter coefficients and signal samples appearing at filter input. Figure 2-5-1 illustrates block diagram of input signal filtering and quantization of result as well.

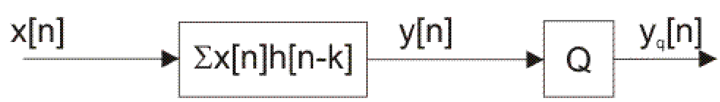


Figure 2-5-1. Signal samples filtering

Multiplication of two numbers each N bits in length, will give a product which is 2N bits in length. These extra N bits are not necessary, so the product has to be truncated or rounded off to N bits, producing truncation or round-off errors. The later is more preferable in practice because in this case the mid-value of quantization error (quantization noise) is equal to 0.

In most cases, hardware used for FIR filter realization is designed so that after each individual multiplication, a partial sum is accumulated in a register which is 2N in length. Not before the process of filtering ends, the result is quantized on N bits and quantization noise is introduced, thus drastically reduced.

Quantization noise depends on the number of bits N. The quantization noise is reduced as the number of bits used for sample and coefficient representation increases.

Both filter realization and position of poles affect the quantization noise power. As all FIR filter poles are located in z=0, the effect of filter realization on the quantization noise is almost negligible.

2.5.3 Overflow

Overflow happens when some intermediate results exceed the range of numbers that can be represented by the given word-length. For the fixed-point arithmetic, coefficients and samples values are represented in the range -1 to +1. In spite of the fact that both FIR filter input and output samples are in the given range, there is a possibility that an overflow occurs at some point when the results of multiplications are added together. In other words, an intermediate result is greater than 1 or less than -1.

Example:

Assume that it is needed to filtrate input samples using a second-order filter.

Such filter has three coefficients. These are: {0.7, 0.8, 0.7}.

Input samples are: { ..., 0.9, 0.7, 0.1, ...}

By analyzing the steps of the input sample filtering process, shown in the table 2-5-3 below, it is easy to understand how an overflow occurs in the second step. The final sum is greater than 1.

FILTER COEFFICIENTS	INPUT SAMPLE	INTERMEDIATE RESULT
0.7	0.9	0.63
0.8	0.7	0.63 + 0.56 = 1.19
0.7	0.1	1.19 + 0.07 = 1.26

Table 2-5-3. Overflow

As the range of values, defined by the fixed-point presentation, is between -1 and +1, the results of the filtering process will be as shown in the table 2-5-4.

FILTER COEFFICIENTS	INPUT SAMPLE	INTERMEDIATE RESULT
0.7	0.9	0.63
0.8	0.7	0.63 + 0.56 - 2 = -0.81
0.7	0.1	-0.81 - 0.07 = -0.88

Table 2-5-4. Overflow effects

As mentioned, an overflow occurs in the second step. Instead of desired value +1.19, the result is an undesirable negative value -0.81. This difference of -2 between these two values is explained in Figure 2-5-2 below.

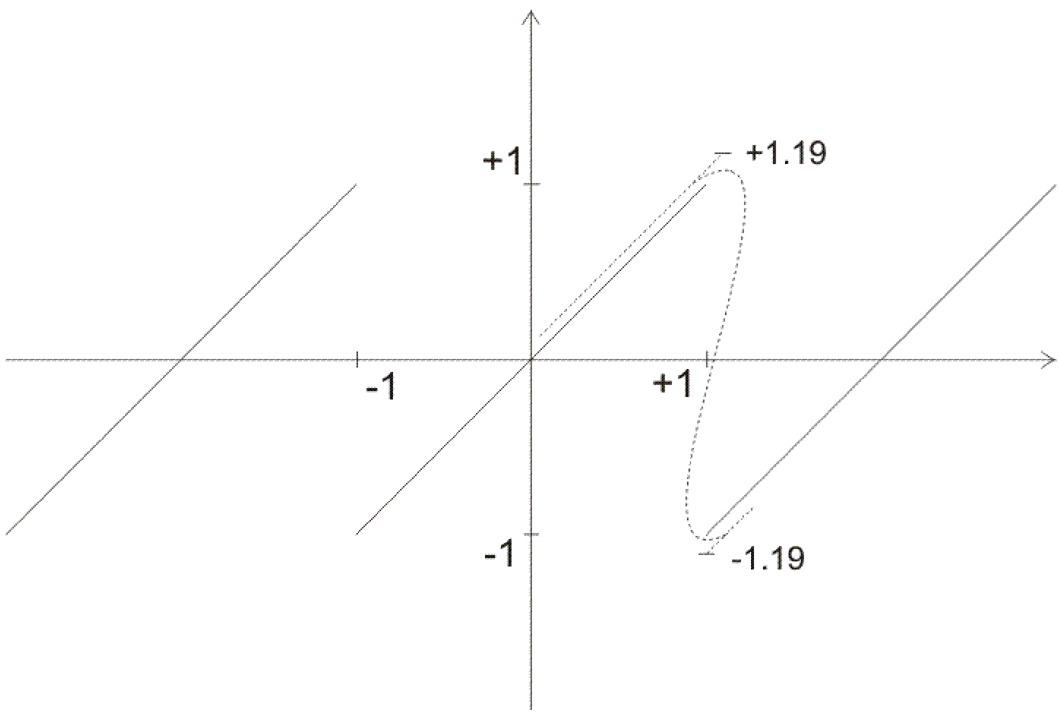


Figure 2-5-2. Signal samples filtering

However, if some intermediate result exceeds the range of presentation, it does not necessarily cause an overflow in the final result. The apsolute value of the result is less than 1 in this case. In other words, as long as the final result is within the word-length, overflow of partial results is not of the essence. This situation is illustrated in the following example.

Example:

The second-order filter has three coefficients. These are: {0.7, 0.8, 0.7}

Input samples are: { ..., 0.9, 0.7, -0.5, ...}

The desired intermediate results are given in the table 2-5-5.

FILTER COEFFICIENTS	INPUT SAMPLE	INTERMEDIATE RESULT
0.7	0.9	0.63
0.8	0.7	0.63 + 0.56 = 1.19
0.7	-0.5	1.19 - 0.35 = 0.84

Table 2-5-5. Desired intermediate results

As seen, some intermediate results exceed the given range and two overflows occur. Refer to the table 2-5-6 below.

FILTER COEFFICIENTS	INPUT SAMPLE	INTERMEDIATE RESULT
0.7	0.9	0.63
0.8	0.7	0.63 + 0.56 - 2 = -0.81
0.7	-0.5	-0.81 - 0.35 + 2 = 0.84

Table 2-5-6. Obtained intermediate results

So, in spite of the fact that two overflows have occured, the final result remained unchanged. The reason for this is the nature of these two overflows. The first one has decremented the final result by 2, whereas the second one has incremented the final result by 2. This way, the overflow effect is annuled. The first overflow is called a positive overflow, whereas the later is called a negative overflow.

Note:

If the number of positive overflows is equal to the number of negative overflows, the final result will not be changed, i.e. the overflow effect is annuled.

Overflow causes rapid oscillations in the input sample, which further causes highfrequency components to appear in the output spectrum. There are several ways to lessen the overflow effects. Two most commonly used are scaling and saturation.

It is possible to scale FIR filter coefficients to avoid overflow. A necessary and sufficient condition required for FIR filter coefficients in this case is given in the following expression:

$$\sum_{k=0}^{N-1} \left| b_k \right| <= 1$$

where:

bk are the FIR filter coefficients; and
N is the number of filter coefficients.

If, for any reason, it is not possible to apply scaling then the overflow effects can be lessened to some extend via saturation. Figure 2-5-3 illustrates the saturation characteristic.

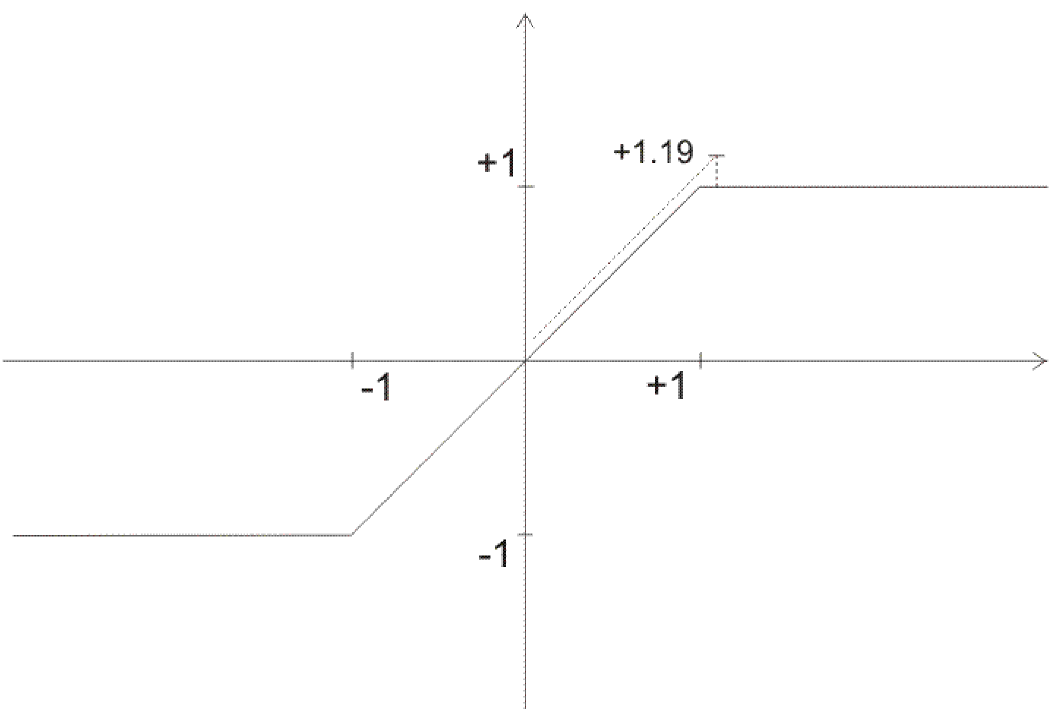


Figure 2-5-3. Saturation characteristic

When the saturation characteristic is used to prevent an overflow, the intermediate result doesn't change its sign. For this reason, the oscillations in the output signal are not so rapid and undesirable high-frequency components are attenuated.

Let's see what happens if we apply the saturation characteristic to the previous example:

Example

Again, it is needed to filtrate input samples using a second-order filter.

Such filter has three coefficients. These are: {0.7, 0.8, 0.7}

Input samples are: { ..., 0.9, 0.7, 0.1, ...}

The desirable intermediate results are shown in the table 2-7-7 below.

FILTER COEFFICIENTS	INPUT SAMPLE	INTERMEDIATE RESULT
0.7	0.9	0.63
0.8	0.7	0.63 + 0.56 = 1.19
0.7	0.1	1.19 + 0.07 = 1.26

Table 2-5-7. Desirable intermediate results

As the range of values, defined by the fixed-point presentation, is between -1 and +1, and the saturation characteristic is used as well, the intermediate results are as shown in the table 2-5-8.

FILTER COEFFICIENTS	INPUT SAMPLE	INTERMEDIATE RESULT
0.7	0.9	0.63
0.8	0.7	0.63 + 0.56 = 1
0.7	0.1	1 + 0.07 = 1

Table 2-5-8. Intermediate results and saturation characteristic

The resulting sum is not correct, but the difference is far smaller than when there is no saturation:

Without saturation: $\Delta = 1.26 - (-0.88) = 2.14$

With saturation: $\Delta = 1.26 - 1 = 0.26$

As seen from the example above, the saturation characteristic lessens an overflow effect and attenuates undesirable components in the output spectrum.

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